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Forecasting the development indicators of oil and gas reservoirs based on initial period data

Prognozowanie wskaźników zagospodarowania złóż ropy naftowej i gazu ziemnego na podstawie danych z okresu początkowego

Vadim O. Bogopolsky, Ramil M. Mammadov

Azerbaijan State Oil and Industry University

ABSTRACT: The practice of oil fields development indicates that the economic efficiency of this process is largely determined by the oil production regime, which is characterized by indicators of the intensity of hydrocarbon extraction: the total number of wells in the field, the dynamics of their commissioning, and production volumes from each of them. At the same time, when justifying oil production projects, it is a very common opinion that it is premature to address the problems of managing the rate of raw material selection in the early stages of project development, often relegated solely to petroleum engineering without consideration of economic aspects. This often leads to a decrease in the efficiency of the entire project due to the inability to control the oil production regime in later stages. This formulation of the problem is particularly relevant for new oil production regions. In well-studied oil and gas region, an error in selecting the rate of raw material production may not be so critical. However, in the case of new fields in undeveloped regions, the cost of such an error can significantly exceed the economic benefits derived from the sale of all the oil produced. In this regard, the problem of developing models and methods for determining economically feasible regimes for oil field development based on controlling the rate of product selection is relevant. The article predicts the dynamics of the following indicators of oil and gas field development: current and cumulative production of oil, water and gas, average well flow rate, and water cut depending on the number of production wells, etc. It demonstrates how it is possible to simplify two- and three-parameter models using the proposed forecasting technique.

Key words: current cumulative oil recovery, cumulative water injection, current oil recovery rate, two-parametrical model, gas-oil ratio (GOR).

STRESZCZENIE: Praktyka zagospodarowania złóż ropy naftowej wskazuje, że o efektywności ekonomicznej tego procesu w dużej mierze decyduje system wydobycia ropy naftowej, który charakteryzują wskaźniki intensywności wydobycia węglowodorów: łączna liczba odwiertów na złożu, dynamika ich uruchamiania oraz wielkość wydobycia z każdego z nich. Jednocześnie przy uzasadnianiu projektów wydobycia ropy naftowej bardzo powszechna jest opinia, że podejmowanie problemów zarządzania wskaźnikiem wydobycia surowca na wczesnych etapach rozwoju projektu jest przedwczesne, a tym samym często sprowadzone wyłącznie do zagadnień z zakresu inżynierii naftowej, bez uwzględnienia aspektów ekonomicznych. Prowadzi to często do spadku efektywności całego projektu ze względu na niemożność kontrolowania reżimu wydobycia ropy naftowej na późniejszych etapach. Takie sformułowanie problemu jest szczególnie istotne w przypadku nowych obszarów wydobycia ropy naftowej. W przypadku dobrze rozpoznanych rejonów wydobycia ropy i gazu błąd w doborze wskaźników wydobycia surowca może nie być aż tak istotny. Jednak w przypadku nowych złóż w niezagospodarowanych regionach, koszt takiego błędu może znacznie przekroczyć korzyści ekonomiczne płynące ze sprzedaży całej wydobytej ropy. W związku z tym istotny jest problem opracowania modeli i metod określania ekonomicznie opłacalnych warunków eksploatacji złóż ropy naftowej w oparciu o kontrolę tempa wyboru wielkości wydobycia. W artykule przedstawiono prognozy dynamiki następujących wskaźników rozwoju pól naftowych i gazowych: bieżącego i skumulowanego wydobycia ropy naftowej, wody i gazu, średniego natężenia przepływu w odwiercie, redukcji udziału wody w zależności od liczby odwiertów wydobywczych itp. Zaprezentowano, w jaki sposób można uprościć dwu- i trzyparametrowe modele za pomocą proponowanej techniki prognozowania.

Słowa kluczowe: bieżące skumulowane wydobycie ropy naftowej, skumulowane zatłaczanie wody, aktualny współczynnik sczerpania ropy, model dwuparametryczny, wykładnik gazowy (GOR).

Corresponding author: V.O. Bogopolsky, e-mail: *vadim46.46@mail.ru*

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Introduction

In recent years, within the oil industry, interest in the problems of constructing mathematical models of oil and gas production processes has surged due to the widespread implementation of information systems, modern technologies for well exploration, systems for recording various information on the state of development objects, wells, oil reservoirs and fields. (Pyankov, 1997; Mirzajanzade et al., 1999; Kostyuchenko and Yampolsky, 2000; Shakhverdiev, 2001; Khurgin, 2004; Sevostyanov and Sergeev, 2004).

The problems of constructing mathematical models of oil and gas production processes are associated with solving identification problems, often referred to as inverse problems.

The identification task entails building optimal, in the sense of given quality criteria, mathematical models of technological development parameters (oil production, liquid, water, reservoir pressure, water cut, etc.) based on field data and the results of comprehensive studies of wells and oil reservoirs.

The tasks of identifying technological parameters are divided into two large areas, each with distinct goals and objectives.

The first area deals with problems at the design level for oil field development, which are solved by large teams in research centers of oil companies and design institutes (Regulations for the preparation of design and technological documents for the development of oil and gas and oil fields, 1996).

At the design stage of field development, digital geological and technological models of oil fields are created, facilitating the prediction of development indicators for a fairly long period (20–30 years), technological schemes and development projects creation, and determination of the company's development strategy.

The second area involves tasks related to identifying the level of monitoring and operational management of oil field development.

At the stage of field development, modeling of technological parameters is crucial for quickly addressing issues such as forecasting oil production, assessing the effectiveness of geological and technical measures, and determining optimal operating conditions for wells (Mirzajanzade et al., 1992).

At this stage, various regression static and dynamic models of technological development parameters are more agile and easily adaptable (customizable) based on field data and well research results compared to geological and technological models at the design level.

Field technology models based on displacement characteristics, fluid filtration equations, and low-parameter regression models of oil, liquid, and water production are widely utilized (Pyankov, 1997; Shakhverdiev, 2001).

The initial period for each reservoir development indicator is defined as the time required for this indicator to reach the value of the so-called "golden section" value, i.e. a value of 38–39% of its final value.

The "golden ratio" is understood as a "section" that divides a whole into two parts in such a way that the smaller (x) part of the whole (1) relates to its larger part in the same way as its larger part relates to the whole itself.

$$\frac{x}{1-x} = \frac{1-x}{1} \tag{1}$$

After solving this equation, we get: $x = 0.385$ (~38.5%) for the smaller part and 0.615 (~61.5%) for the larger part of the whole.

It is known that the primary stage of production, that is, the initial period is ~38–39% of its final value, which coincides with the value of the "golden ratio".

Therefore, in the article, this period is conventionally called the "golden section", and the time from the start of field development until oil recovery reaches its "golden section" is called the initial period for the "current accumulated oil recovery" indicator.

In the work, based on the analysis of actual data of a 20–30 year history of the development of two dozen oil and gas fields in Transcaucasia, the North Caucasus and the Volga region (Ivanova, 1976), a certain regularity was established. It was found that when certain indicators (as a function of time) reach their "golden section", the rate of change of this indicator, that is, its first derivative with respect to time reaches a maximum and its second derivative becomes zero. The article demonstrates how, utilizing this regularity and extremely simplified two- and three-parametric mathematical models for similar objects, one can reliably predict reservoir development dynamics while considering regulators such as well grid density, production withdrawal rate, and water injection rate.

In this context, it turns out that the initial periods for each specific reservoir development indicator do not coincide in time with each other.

For example, maximum current oil production rates at the end of the initial period for this indicator, as a rule, precede the maximum increase in water cut by 2–3 years.

For example, the current cumulative oil recovery of a reservoir at the level of 38–39% of its final value is considered the "golden section" for oil recovery, and the time from the start of development of the object until oil recovery reaches its "golden section" is called the initial period for the "current cumulative oil recovery" indicator.

The same applies to the indicator of the current water-cut of the product and other indicators.

It is assumed that the rate of oil production at an object is higher when the remaining recoverable oil reserves at the object are greater and when the reservoir properties of the object are better.

The parameters characterizing the reservoir properties of the object are determined, and it is shown that the higher the permeability, porosity, oil saturation and homogeneous nature of the formation, the lower the oil viscosity and the greater the oil recovery factor of the formations.

It is further demonstrated that the average current oil production per one operating production well of an object depends on the properties of the produced product, reservoir properties of the object, the number of active production wells at the object, and the recoverable oil reserves per one production well, and can be implemented in a simplified two-parameter form.

The next stage involves the development of an exchange program enabling the regulation of oil selection during fluctuating water cuts.

About logistics *S*-shaped models

Figure 1 shows the smoothed author's model illustrating the logistic *S*-shaped dynamics of the reserve $Q(t)$ [quantity] extracted from the beginning of the development of a certain geological object and the final recoverable reserve of the object for the implemented technology $Q(t = \infty) = Q^*$. For symbols, refer to the legend below.

Figure 1 shows the smoothed model (in intervals of a month, quarter, year) proposed by the authors, illustrating the dynamics of the rate of extraction of the object's reserves $q(t)$; its dimension [quantity/time step δt] (oil, gas, coal, ore, field as a whole, horizon, group of wells, etc.).

The typical form of the indicated *S*-shaped logistic model curve $Q(t)$ is consistent with the data in the work of Ivanova (1976) and is confirmed by numerical comparisons of the model and actual results presented in Tables 1 and 2 of this article.

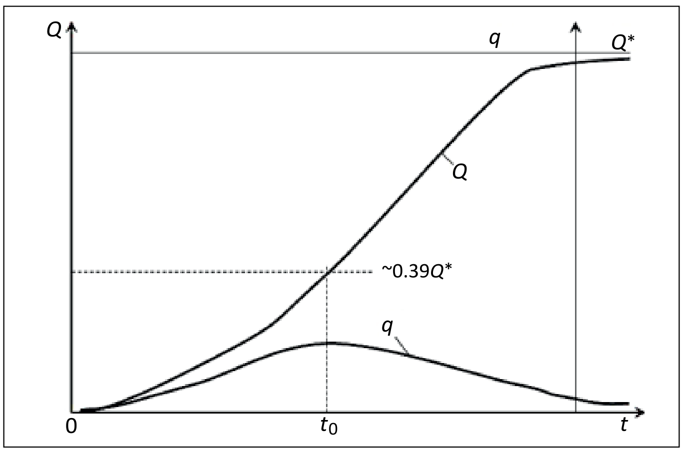

Figure 1. Authors' model

Rysunek 1. Model opracowany przez autorów

For comparison, Figure 2 with the index *f* below, presents the well-known logistic *S*-shaped model of Verhulst–Pearl (Verhulst, 1838, 1845; Pearl and Lowell, 1920; Kingsland, 1982), often used in problems of the growth of biological populations, statistics, etc.

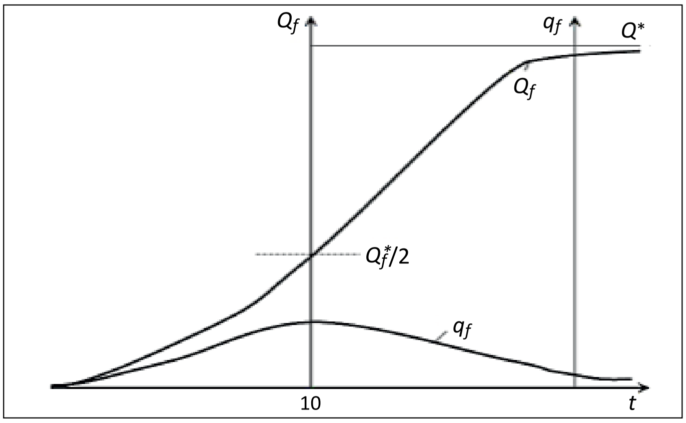

Figure 2. Verhulst–Pearl model (Verhulst, 1838, 1845; Pearl and Lowell, 1920)

Rysunek 2. Model Verhulsta-Pearla (Verhulst, 1838, 1845; Pearl and Lowell, 1920)

The idea of using the Verhulst–Pearl model in problems of developing oil and gas fields was proposed for the first time in the work of Mirzajanzade et al. (1992). In this article, the effectiveness of this concept was practically confirmed across 20 oil and gas facilities by replacing the Verhulst–Pearl model with the proposed *S*-shaped model.

The characteristic mathematical features and differences of both models and their respective *S*-shaped curves are indicated below. Definition areas:

Verhulst–Pearl model:

$$-\infty \leq t \leq \infty;\ Q_f(-\infty) = 0 \leq Q_f(t) \leq Q_t^* = Q_f(\infty);$$

authors' model:

$$0 \leq t \leq \infty;\ Q_f(0) = 0 \leq Q(t) \leq Q^* = Q(\infty).$$

In Figures 1 and 2, the points $Q(t_0)$ and $Q_f(0)$ represent the inflection points of the $Q(t)$ and $Q_f(t)$ curves, from which the final extractions Q^* and Q_f^* are predicted in the models.

The inflection point changes and the final extractable resource changes accordingly.

Therefore, in the proposed model, the inflection point is $Q(t_0) = Q^*/0.39$, and t_0 is a variable parameter, while in the Verhulst–Pearl model, the inflection point is always $Q_f(0) = Q_f^*/2$, which is its disadvantage.

The second significant drawback of the Verhulst–Pearl model is the necessity to operate with the commencement of work on the site from an uncertain point in time $t = -\infty$, leading to deliberately arbitrary and therefore unreliable results in estimating the value of $Q_f(0)$ and, consequently, the predicted value of Q_f^*.

In dimensionless variables, the *S*-shaped models of Verhulst–Pearl and the authors are presented below.

$$\frac{dZ_f}{d\tau} = Z_f(1-Z_f);\ Z_f = \frac{1}{\left[1+\exp(-\tau)\right]};$$
$$\tau = A_f t;\ Z_f = \frac{Q_f(t)}{Q_f^*} \quad (2)$$

$$\frac{dZ}{d\tau^2} = (1-Z);\ Z = 1-\exp(-\tau^2);\ \tau = At;\ Z_f = \frac{Q(t)}{Q^*} \quad (3)$$

Methodology

In this work, the following designations of dimensional and dimensionless indicators are used:

- $Q_{o,w,g}(t)$ – current cumulative production (extraction) of oil, water, gas in the field from the start of development, (t);
- $Q^*_{o,w,g}(t)$ – expected final cumulative production (extraction) of oil, water, gas for the object (t);
- $q_{o,w,g}(t) = dQ_{o,w,g}(t)/dt$ – current production of oil, water, gas at the field (t/year, t/quarter, etc.);
- $b(t) = q_w/(q_w + q_o)$ – current water-cut of the object;
- $w(t) = q_w/q_o$ – current water-oil ratio for the object;
- $B(t)$, $W(t)$ – cumulative values, with $B^*(t)$, $W^*(t)$ – cumulative values at the end of the development;
- $Z(t) = Q_o(t)/Q_o^*(t)$ – current oil recovery from the final recoverable oil reserves;
- $z(t) = q_o(t)/Q_o^*(t)$ – current rate of oil production from the final recoverable oil reserves;
- g – gas-oil ratio, GOR [m^3/t];
- A – a parameter characterizing the reservoir properties of the object (dimension A is inverse to the dimension of time, t);
- $n(t)$ – the current annual operating fund of production wells of the object, with N^* – all production wells of the field that have been active during the development period;
- $Q(t)$, $q(t)$ – cumulative water injection (t) and the current rate of its injection over the reservoir (t/year, t/quarter, etc.); the same designations of indicators with the index s below, for example, $Q_{so}(t)$, $q_{so}(t)$, b_s, A_{so} – the average value of this indicator for one well.

1. It is assumed that the rate of oil production at an object is higher when there are greater residual recoverable oil reserves at the reservoir and when the reservoir properties of the object are better.

   A two-parametric mathematical model for an object, proposed by the author, is presented below in both dimensional and dimensionless forms:

$$q_o(t) = \frac{dQ_o(t)}{dt} = 2A_o^2 t(Q_o^* - Q_o) = 2A_o^2 t Q_o^* \exp[-(A_o t)^2]$$
$$q_o(0) = q_o(\infty) = 0$$
$$z_o(t) = \frac{dZ_o}{d\tau^2} = 1 - Z_o = \exp(-\tau^2)$$
$$\tau = A_o t,\ \frac{dZ_o(0)}{d\tau^2} = 1,\ \frac{dZ_o(\infty)}{d\tau^2} = 0 \quad (4)$$

$$Q_o(t) = Q_o^*\left\{1-\exp[-(A_o t)^2]\right\};\ Q_o(0) = 0;\ Q_0(\infty) = Q_o^*$$
$$Z_o(t) = 1-\exp(-\tau^2);\ Z_o(0) = 0,\ Z_o(\infty) = 1 \quad (5)$$

The *S*-shaped curve (5) $Q_o(t)$ at $t = t_0$ has an inflection point at which its second derivative is zero.

$$\frac{d^2Q_o(t_{0_o})}{dt^2} = 2Q_o^* A_o^2 \exp\left[-(At_{0_o})^2\right]\left[1-2(A_o t_{0_o})^2\right] = 0$$
$$\frac{d^2Z}{d\tau^2} = 2\exp(-\tau^2)\left[1-2\tau^2\right] = 0 \quad (6)$$

oil production rate reaches its maximum

$$q_o(t_{0_o}) = \frac{dQ_o(t_{0_o})}{dt} = \max(t) = 1.5415\frac{Q_o(t_{0_o})}{t_{0_o}} 0.6065\frac{Q_o^*}{t_{0_o}},$$
$$z_o(t_{0_o}) = \frac{0.6065}{(t_{0_o})} \quad (7)$$

$$Z(t_0) = \frac{Q_o(t_{0_o})}{Q_o^*} = 0.3935 \text{ or } \sim 39\% \quad (8)$$

$$t_{0_o} = \frac{1}{A_o\sqrt{2}} = \frac{0.7071}{A_o};\ \tau_{0_o}^2 = \frac{1}{2} \quad (9)$$

The time $t = t_{0_o}$ is the end of the so-called initial object development period, when the current oil recovery reaches its "golden section" at ~39% of the final oil extraction of the reservoir Q_o^*, the oil recovery rate reaches its maximum equal to ~61/t_o(%) of the final oil recovery Q_o^*, and the parameter A_o characterizes the reservoir properties of the object, such as the higher the permeability, porosity, oil saturation, and uniformity of the reservoir in thickness and extent, the lower the viscosity of oil, etc.

The presence of only these two dependencies (8) and (9) enables the prediction of the final recoverable oil reserves from the point t_0 by the values of $Q_o(t_{0_o})$, $q_o(t_{0_o})$ known at this point, like:

$$Q_o^* = \frac{Q_o(t_{0_o})}{0.3935} = \frac{q_o(t_{0_o})t_{0_o}}{0.6065} \quad (10)$$

Model (4)–(9) is refined and adjusted by an independent condition:

$$(A_o t_0)^2 = \frac{1}{2} = 0.3234\frac{q_o(t_0)t_0}{Q_o(t_0)} \quad (11)$$

in which all its members $dQ_o(t_{0_o})/dt$, $Q_o(t_{0_o})$ and t_{0_o} are known de facto. Thus, the condition (10) specifies the end time of the initial-production period.

Therefore, both unknown parameters of the model $Q_o^* = 2.5415Q_o(t_{0_o})$, A_o and all subsequent dynamics of indicators (4), (5) of the development of an oil and gas field for recoverable oil can be represented, for example, in the form:

$$Z(t) = 1 - \exp\left(\frac{-(t/t_{0_o})^2}{2}\right);$$

$$z(t) = \frac{dZ(t)}{dt} = (t/t_{0_o})^2 \exp\left(\frac{-(t/t_{0_o})^2}{2}\right) \qquad (12)$$

2. The current water production, denoted as $b(t)$, is also described by an S-shaped two-parameter model, similar to (4)–(12). Moreover, in (4)–(12), $Z_o(t,\tau)$ and A_o should be replaced by $b(t,\tau)$ and A_b, respectively.
It should be noted that $A_b \neq A_o$, and therefore, the initial periods for water-cut and oil recovery, $t_{0b} \neq t_{0o}$, will also differ.

$$b(t) = 1 - \exp[-(A_b t)^2];\; b(0) = 0,\; b(\infty) \to 1$$

$$b(t_{0_b}) = 1 - e^{-\frac{1}{2}} = 0.3935 \quad \text{or} \quad \sim 39\%$$

$$(A_b t_{0_b})^2 = \frac{1}{2} 0.3144 \frac{\frac{db(t_{0_b})}{dt} t_{0_b}}{b(t_{0_b})} \qquad (13)$$

Current water-oil ratio (WOR):

$$w(t) = \frac{q_w(t)}{q_o(t)} = \frac{b(t)}{1 - b(t)} = \exp(A_b t)^2 - 1;$$
$$w(0) = 0,\; w(\infty) \to \infty \qquad (14)$$

Current water production rate:

$$q(w) = w(t) q_o(t) = Q_o^* \left(\frac{t}{t_{0_o}^2}\right)\left[\exp(A_b t)^2 - 1\right]\exp(-A_o t)^2$$
$$q_w(0) = 0,\; q_w(\infty) \to \infty \; if \; A_o \prec A_b, t_{0_o} \succ t_{0_b},$$
$$q_w(\infty) \to 0 \; if \; A_o \succ A_b, t_{0_o} \prec t_{0_b} \qquad (15)$$

Cumulative water production:

$$Q_w(t) = \int q_w(t) dt =$$
$$= Q_o^* \left\{\left[\frac{k - \exp\left(-(1-k)\lambda^2 \frac{(t)}{2}\right)}{(1-k)}\right] + \exp\left(-\frac{\lambda^2(t)}{2}\right)\right\};$$
$$k = \left(\frac{t_{0_o}}{t_{0_b}}\right)^2;\; \lambda(t) = \frac{t}{t_{0_o}};\; Q_w(0) = 0,\; Q_w(\infty) \to \infty \text{ if } k \succ 1,$$
$$Q_w(\infty) \to \frac{k}{1-k} \text{ if } k \prec 1 \qquad (16)$$

Cumulative water-oil ratio:

$$W(t) = \frac{Q_w(t)}{Q_o(t)} =$$
$$= \frac{k - \left[\exp\left(-(1-k)\frac{\lambda^2(t)}{2}\right)\right] + (1-k)\exp\left(-\frac{\lambda^2(t)}{2}\right)}{(1-k)\left[1 - \exp\left(-\frac{\lambda^2(t)}{2}\right)\right]}$$
$$W(0) = 0,\; W(\infty) \to \infty \text{ if } k \succ 1,\; W(\infty) \to \frac{k}{1-k} \text{ if } k \prec 1 \qquad (17)$$

3. The amount of reservoir gas produced with oil is predicted based on the amount of produced oil multiplied by the oil-gas ratio:

$$q_g(t) = g(t) q_o(t)$$

The oil-gas ratio of the reservoir depends on the initial amount of gas dissolved in oil $g(0)$ and the current reservoir pressure, P. Works on maintaining reservoir pressure keep the oil-gas ratio fairly stable ($g(t)$ = const); otherwise, it falls in proportion to the pressure drop, and the latter is proportional to the residual oil reserves (Aziz and Settari, 1982; Barenblatt and Ryzhik, 1984; Aziz, 1994).

$$g(t) = \frac{g(0)P(t)}{P(0)} = g(0)\left(1 - Z(t)\right) \qquad (18)$$

4. The average current oil production per operating production well of an object depends on the properties of the produced products, the reservoir properties of the formation, the number of operating producing wells at the field, recoverable oil reserves per production well and in a simplified two-parametric form:

$$q_{so}(t) = \frac{q_o(t)}{n(t)} = q_{so}(0)\exp\left(-(A_{so}t)^2 - (Ct)^2\right);$$
$$q_{so}(0) = \left(\frac{A_o}{C_0}\right)^2 \left(\frac{Q_o^*}{N^*}\right);$$
$$q_o = \frac{dQ_o(t)}{dt} = 2A_o^2 t Q_o^* \exp\left(-(A_o t)^2\right);$$
$$n = \frac{dN(t)}{dt} = 2C_0^2 t N^* \exp\left(-(Ct)^2\right);$$
$$Q_o(t) = Q_o^*\left[1 - \exp\left(-(A_o t)^2\right)\right];$$
$$N(t) = N^*\left[1 - \exp\left(-(Ct)^2\right)\right] \qquad (19)$$

Interesting from a practical point of view is the case in which the current fund of operating producing wells of the field increases to a maximum, and then stabilizes and decreases:

$$C = C(t) = (C_0 - C_{00})\left(\exp(-mt)\right) + C_{00} = (C_0 - C_{00})\eta^{t/ts} + C_{00},$$
$$C(t_s) = A_o,\; m = \left(\frac{1}{t_s}\right)\ln\left(\frac{1}{\eta}\right),\; \eta = \frac{(A_o - C_{00})}{(C_0 - C_{00})} \prec 1,\; C_{00} \prec A_o \prec C_0 \qquad (20)$$

In this case:

- on the interval $t > t_s$, $dq_{so}(\infty)/dt = q_{so}(\infty) \to 0$, $C(t) < A_n$, $q_{so}(t) < q_{so}(0) = q_{so}(t_s)$;
- on the interval $t(0, t_s)$, $q_{so}(t) > q_{so}(0)$, while at its ends $q_{so}(t_s) = q_{so}(0)$;
- and in the middle $q_{so}(t)$ has a maximum:

$$q_{so}\left(\frac{t_s}{2}\right) = q_{so}(0)\exp\left[-\left(A_o^2 - C_0^2\left((1-v)\eta^{\frac{1}{2}} + v\right)^2\right)\frac{t_s^2}{4}\right],$$

$$v = \frac{C_{00}}{C_0} \tag{21}$$

Relations (19)–(21) mean that the coincidence of the initial periods of recoverable oil for the reservoir as a whole and for the average oil flow rate per operating production well of the reservoir, though not excluded, is usually absent.

In particular, in the work of Ivanova (1976), such a coincidence occurred in 4 out of 20 reservoirs; in these cases $C = \text{const}(t) < A_o$, but on the remaining reservoirs $C = C(t)$, and the indicated coincidence was absent.

In the model (19)–(21), there are two unknowns and three parameters known de facto.

The values of $q_{so}(0)$ and t_s are established by the actual oil production of the first wells of the object and the time of the end of the period when $q_{so}(t) > q_{so}(0)$. The value of A_o is set by the initial period of the current oil production $q_o(t)$.

Two unknown parameters C_0, C_{00} are determined from two conditions (20), (21) at the point $t_s/2$ by the known max $q_{so}(t_s/2)$ (18) and at the point t_s, in which $C(t_s) = A_o$ (17).

5. Prediction of the effectiveness of the impact on the reservoir and the CCD requires a preliminary forecast of the indicators of the basic variant of reservoir development without the impact of c, etc., using models (4)–(16). Then, using the same models (4)–(16), additional extraction of oil, water and gas is predicted based on the results of the initial exposure period (Kanevskaya, 2002), such as the difference between the current additional oil, water, and gas extraction during the exposure period and the basic production option n, w and g on the reservoir without exposure:

$$\Delta Q_o(\Delta t) = Q_o(\Delta t) - Q_{ox}(\Delta t),\ \Delta q_o(\Delta t) = q_o(\Delta t) - q_{ox}(\Delta t), \text{ etc.}$$

Here $\Delta t = t - t_x$, where t is the time from the beginning of the development of the reservoir and t_x is the time of the beginning of the impact on the reservoir.

Calculations are based on the actual maximum of the additional extracted oil and water, achieved at the field due to the impact at the end of the initial exposure period.

$$\Delta Q_o(\Delta t_{0_o}) = Q_o(\Delta t_{0_o}) - Q_{xo}(\Delta t_{0_o}) = 0.3953\Delta Q_o^*;$$

$$(\Delta A_o \Delta t_{0_o})^2 = 1/2 \tag{22}$$

Model calculations and comparison with actual results of oil development

A comparison of actual (Ivanova, 1976) and model calculations for cumulative recovery (Z) and the annual rate of oil withdrawal (z) in the initial period (t_{0n}) and at the end of development (t) are presented in Table 1.

A comparison of actual (Ivanova, 1976) and model calculations of the current water cut (b) and the accumulated water-oil factor (W) at the end of the initial period (t_{b0}) and at the end of the development of the object (t) are presented in Table 2.

Discussion of results

The word "model" in Tables 1 and 2 marks the columns that present the results of model calculations of current (at time t) annual and accumulated since the beginning of development ($t = 0$) selections. To compare this calculated indicator with the actual one, the value of the latter at the same point in time is indicated from the corresponding table in the book (Ivanova, 1976).

All calculations presented in Tables 1 and 2 of the article were made using the formulas written under the tables. These are the same model formulas proposed above, but instead of the value $\tau = At$, the identical $t/t_0\sqrt{2}$ is used in them.

Table 1. Comparison of actual (Ivanova, 1976) and model calculations for cumulative extraction (Z) and annual rate of oil recovery (z) in the initial period (t_{0n}) and at the end of development (t)

Tabela 1. Porównanie rzeczywistych (Ivanova, 1976) i modelowych obliczeń dla skumulowanego wydobycia (Z) i rocznego wskaźnika wydobycia ropy (z) w początkowym okresie (t_{0n}) i pod koniec eksploatacji (t)

Reservoir /1/		t_{0n}	$Z(t_{0n})$		max $z(t_{0n})$		t	$Z(t)$		$z(t)$	
		[year]	[%]		[%/year]		[year]	[%]		[%/year]	
sheet	table	actual	actual	model	actual	model	actual	actual	model	actual	model
32	6	8	**39.4**	**39.0**	11.4	7.7	16	98.5	86.5	0.5	3.0
34	**7**	6	**38.5**	**39.0**	5.9	10.1	15	**84.4**	**87.5**	2.7	1.8
36	**8**	9	**40.1**	**39.0**	**5.9**	**6.7**	23	79.9	96.2	**1.1**	**1.1**

cont. Table 1/cd. Tabela 1

Reservoir /1/		t_{0n} [year]	$Z(t_{0n})$ [%]		max $z(t_{0n})$ [%/year]		t [year]	$Z(t)$ [%]		$z(t)$ [%/year]	
sheet	table	actual	actual	model	actual	model	actual	actual	model	actual	model
64–65	**14**	9	**39.3**	**39.0**	**6.7**	**6.8**	24	87.4	97.1	**0.4**	**0.5**
67	15	7	**40.6**	**39.0**	–	–	23	82.5	99.5	–	–
70	16	12	**39.6**	**39.0**	**5.5**	**5.1**	26	82.2	90.4	5.5	1.7
84–85	**19**	10	**40.0**	**39.0**	**6.3**	**6.0**	26	89.5	96.6	**0.3**	**0.5**
94	**22**	10	**38.5**	**39.0**	**6.2**	**6.1**	25	88.0	95.6	**1.3**	**1.1**
97	**23**	10	**41.3**	**39.0**	–	–	25	**95.6**	**95.6**	–	–
101	**24**	11/12	**38.9**	**39.0**	**4.8**	**5.0**	21	**79.0**	83.8/**78.3**	**3.5**	2.8/**3.3**
102–103	**25**	11	**38.8**	**39.9**	**4.9**	**5.6**	25	84.5	92.4	**2.2**	**1.8**
106–107	**26**	11/12	**39.0**	**39.0**	**4.4**	**5.0**	24	**86.5**	90.1/**86.5**	3.6	1.8/2.2
112–113	27	8/9	**40.0**	**39.0**	**7.3**	**7.8**	25	90.0	99.2/98.4	2.5	0.3/0.7
116–117	**28**	**8/9**	**40.0**	**39.0**	10.5	7.8	26	**95.7**	99.4/**98.4**	1.1	0.2/0.5
118	29	7/8	**38/39**	**39.0**	**7.5**	**8.1**	22	87.7	99.3/97.7	2.0	0.3/0.8

To the Table 1: $\max z(t_{0_o}) = \dfrac{0.6065}{t_{0_o}}$; $z(t) = \left(\dfrac{t}{t_{0_o}}\right)^2 \exp\left(-\dfrac{(t/t_{0_o})^2}{2}\right)$; $Z(t) = 1 - \exp\left(-\dfrac{(t/t_{0_o})^2}{2}\right)$

Table 2. Comparison of actual (Ivanova, 1970, 1976) and model calculations of the current water cut (b) and cumulative water-oil ratio (W) at the end of the initial period (t_{b0}) and at the end of the development of the object (t)

Tabela 2. Porównanie rzeczywistych (Ivanova, 1970, 1976) i modelowych obliczeń bieżącego udziału wody (b) i skumulowanego stosunku woda/olej (W) pod koniec okresu początkowego (t_{b0}) i pod koniec eksploatacji obiektu (t)

Reservoir /1/		t_{0n} [year]	$b(t_{b0})$ [%]		$b(t)$ [%]		t [year]	k	λ^2	$W(t)$	
sheet	table	actual	actual	model	actual	model	actual			actual	model
32	6	14.0	39.0	39.0	76.3	49.3	16	0.57	7.11	0.12	0.87
34	7	14.0	33.4	39.0	28.3	43.7	15	0.42	6.25	**0.15**	**0.46**
36	**8**	13.0	**39.0**	**39.0**	**79.2**	**79.0**	23	0.64	6.53	**0.65**	**1.05**
55	11	21.0	37.6	39.0	88.0	64.0	30	–	–	1.01	–
57	**12**	15.5	**40.0**	**39.0**	**89.5**	**78.1**	27	–	–	**1.12**	–
59	13	21.5	39.9	39.0	89.5	64.5	31	–	–	0.86	–
64–65	14	13.5	34.0	39.0	95.0	82.0	25	0.67	7.72	**1.15**	**1.19**
67	**15**	13.0	**38.0**	**39.0**	**91.0**	**91.8**	24	0.54	11.75	**1.09**	**1.03**
70	16	16.0	34/42	39.0	87.4	73.3	26	0.75	4.69	**0.69**	**0.86**
75	17	12.0	39.3	39.0	87.0	75.1	20	–	–	0.62	–
81	**18**	20.0	**38.3**	**39.0**	**75.5**	**75.5**	29	–	–	–	–
84–85	19	18.0	39.0	39.0	91.4	64.8	26	0.55	6.76	**0.44**	**0.35**
94	**22**	12.0	**41.7**	**39.0**	**89.7**	**88.6**	25	0.83	6.25	–	–
97	**23**	13.0	**42.8**	**39.0**	**95.3**	**84.3**	25	0.77	6.25	–	–
101	**24**	15.0	**40.0**	**39.0**	**68.7**	**62.5**	21	0.73	3.64	**0.50**	**0.66**
102–103	**25**	9.0	32.0	39.0	**87.5**	**97.8**	25	1.22	5.16	**2.26**	**2.60**
106–107	**26**	18.0	40.1	39.0	**61.3**	**59.0**	24	0.66	4.00	**0.49**	**0.57**
112–113	27	9.0	40.0	39.0	**45.0**	**97.8**	25	1.00	9.76	**0.27**	–
116–117	**28**	15.0;	**38.0**	**39.0**	**75.5**	**77.7**	26	0.53	10.56	0.44	0.97
118	**29**	13.0	**38.0**	**39.0**	**87.7**	**81.4**	22	0.58	8.60	0.50	1.07

To the Table 2: $b = 1 - \exp\left(-\dfrac{(t/t_{b0})^2}{2}\right)$; $W = \left[k - \left(\exp\left(-(1-k)\dfrac{\lambda^2}{2}\right)\right) + (1-k)\exp\left(-\dfrac{\lambda^2}{2}\right)\right] \Big/ \left[(1-k) - \left(1 - \exp\left(-\dfrac{\lambda^2}{2}\right)\right)\right]$

In addition, the value $z = q/Q^*$ in the dimension [1/year] (as in formulas (1)–(6) of the article) in Tables 1 and 2 is expressed in the dimension $z = (100 \cdot q)/Q^*$ [%/year], in accordance with what is accepted in (Ivanova, 1976).

In these tables, bold highlighted positions indicate where the coincidence of the actual value of the indicator and its model calculation can be considered satisfactory.

This primarily relates to the actual and model values of $Z(t_{0o})$ and max $z(t_{0o})$ from Table 1.

As observed, for all 15 objects (Table 1) examined at the end of the so-called initial development period, the cumulative oil recovery reached its "golden section" at 39% of the final recoverable reserves, and the current annual rate of oil extraction actually reached its annual maximum at the given reservoir. The only difference was that the actual value of this very annual maximum did not always coincide with its calculated model maximum.

Poor compliance for 5–6 objects out of 15 in Table 1 can be explained by the following reasons:

- insufficient number of downhole instrumentation equipment at the fields, resulting in products not being measured, but distributed among wells and objects planned tasks with large inaccuracies, based on reservoir oil and water production;
- in the tables, the figures are presented in 1-year increments, while in fact, neglecting the indicator for 0.5 years at the end of 10 years gives only a linear error of at least 5%, in 1-year increments – 10% and higher, and in the presented calculations time enters the exponent nonlinearly, increasing this error further.

A comparison of actual (Ivanova, 1970, 1976; Khalimov and Ivanova, 1980) and model calculations for the current water flow and the cumulative water-oil ratio are presented in Table 2.

Satisfactory coincidence of the actual (Ivanova, 1970, 1976; Khalimov and Ivanova, 1980) and estimated model water cut and cumulative water-oil ratio with an error not higher than 5–10% for the initial development period (39%) is observed in almost all 20 sites, and the forecast with the same error for the period of 20–30 years is only in 10 cases out of 20. The reason and conclusions are the same as in the case of oil extraction, but with the amendment that the accuracy of control and metering of the produced water at the indicated fields was set 2-3 times worse than the control of the produced oil.

Conclusions

1. The proposed mathematical models should preferably be used on field data in 1-quarter increments.
2. The models easily identify field problems associated with the lack of downhole instrumentation equipment, forced inaccuracies in the distribution of products among objects according to the "reservoir park", where production from several fields is collected without breakdown by facilities, unrecorded oil and water flows inside well, and perforation of additional intervals, among other issues.
3. The ideal approach for forecasting and managing the development of an oil object is the option of development, arrangement and control, in which each well and all the geological and technical measures carried out on it are designed as an independent object.
4. This is particularly relevant for oil and gas wells in shale reservoirs.

The proposed mathematical models of oil production and identification algorithms allow for:

- Incorporating additional a priori data and expert assessments of technological parameters for field development, such as recoverable reserves, predicted oil production values, oil production model parameters, etc.
- Obtaining estimates of the forecast of oil production and recoverable reserves under conditions of a priori uncertainty about the statistical characteristics of errors in additional a priori information and expert estimates.
- Significantly increasing the accuracy of forecast estimates of oil production and recoverable reserves, by a factor of two or more, with a small volume of field data at the early (first) stage of field development during the first five years, compared to estimates of the Gauss–Newton method, where a priori information is not available taken into account.

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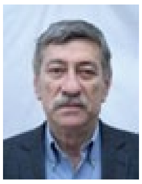
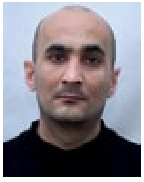


Vadim Oskarovic BOGOPOLSKY, Ph.D.
Assistant professor at the Department of Oil and Gas Engineering
Azerbaijan State Oil and Industry University
16/21 Azadliq Ave, AZ1010, Baku, Azerbaijan
E-mail: *vadim46.46@mail.ru*



Ramil Mukhtar MAMMADOV, M.Sc.
Assistant at the Department of Oil and Gas Engineering
Azerbaijan State Oil and Industry University
34 Azadliq Ave, AZ1010, Baku, Azerbaijan
E-mail: *ramil.mammadov.mu@asoiu.edu.az*