NAFTA-GAZ

Nafta-Gaz 2024, no. 8, pp. 495–500, DOI: 10.18668/NG.2024.08.04

Diagnosis of the formation damage process using the theory of catastrophe

Ocena procesu uszkodzeń formacji z wykorzystaniem teorii katastrof

Ramil M. Mammadov, Arzu V. Sultanova

Azerbaijan State Oil and Industry University

ABSTRACT: A broad area of interdisciplinary research, commonly referred to as nonlinear science, includes nonlinear thermodynamics, catastrophe theory, dynamic chaos theory, and fractal mathematics. This field has produced numerous influential figures, many books, and countless articles. Many popular books have been published on the theory of catastrophes, chaos, and fractals. Systems studied by nonlinear science are usually called complex; their properties are not reducible to those of their components and exhibit newly emerging, or "emergent" features. A catastrophe is an abrupt change that occurs as a sudden response of a system to a gradual change in external conditions. The mathematical description of phenomena associated with sharp and qualitative changes is provided by the theories of singularities and bifurcations. Bifurcations (catastrophes) are discontinuities in systems described by smooth (continuous) functions. This article attempts to make an adaptive operational decision aimed at preventing complications arising from sand production in wells, based on the application of catastrophe theory and mathematical and statistical tools. The influence of sand production on well productivity, particularly the associated plug formation, is a relevant issue and pertinent research focus today. The physical cause of the sand plug's effect is that due to the small cross-section of the wellbore, the sand plug located above the productive horizon acts as a downhole fitting, creating significant resistance to the upward flow. Early diagnosis of the onset of near-wellbore zone damage and the transition of wells to sand-producing status is crucial. In addition, timely control of complications related to sand removal during the complete distortion of the bottomhole zone, when the sand production process is fully established, is of great relevance. The effectiveness of measures to prevent the complications in operation process depends on the validity and efficiency of such control.

Key words: bottomhole zone, sand removal, dynamic system, equilibrium point, equilibrium stability, elementary catastrophe, diagnosis, performance.

STRESZCZENIE: Szeroki obszar badań interdyscyplinarnych, powszechnie określany jako nauka nieliniowa, obejmuje nieliniową termodynamikę, teorię katastrof, dynamiczną teorię chaosu i matematykę fraktalną. Dziedzina ta zaowocowała wieloma wpływowymi postaciami, książkami i niezliczonymi artykułami. Opublikowano wiele popularnych książek na temat teorii katastrof, chaosu i fraktali. Systemy badane przez nauki nieliniowe są zwykle nazywane złożonymi; ich właściwości nie dają się zredukować do właściwości ich komponentów i charakteryzują się nowo powstającymi cechami. Katastrofa to gwałtowna zmiana, która występuje jako nagła reakcja systemu na stopniową zmianę warunków zewnętrznych. Matematyczny opis zjawisk związanych z szybkimi i jakościowymi zmianami zapewniają teorie osobliwości i bifurkacji. Bifurkacje (katastrofy) są nieciągłościami w systemach opisanych przez ciągłe funkcje. W artykule podjęto próbę sformułowania adaptacyjnej decyzji operacyjnej mającej na celu zapobieganie komplikacjom, wynikającym z procesu piaszczenia w odwiertach, w oparciu o zastosowanie teorii katastrof oraz narzędzi matematycznych i statystycznych. Wpływ piaszczenia na produktywność odwiertu, a w szczególności na związane z tym tworzenie się korka piaskowego, jest obecnie istotnym zagadnieniem i przedmiotem badań. Efekt piaszczenia polega na tym, że ze względu na mały przekrój otworu wiertniczego, korek piaskowy znajdujący się nad horyzontem produkcyjnym działa jak armatura wiertnicza, wytwarzając znaczny opór podczas przepływu w górę. Kluczowe znaczenie ma wczesna diagnostyka uszkodzeń strefy przyodwiertowej i przejście odwiertu w stan piaszczenia. Ponadto, bardzo istotna jest terminowa kontrola komplikacji związanych z usuwaniem piasku przy całkowitym zniekształceniu strefy przyodwiertowej, gdy proces piaszczenia jest już w pełni rozwinięty. Skuteczność środków zapobiegających komplikacjom w procesie eksploatacji zależy od zasadności i skuteczności takiej kontroli.

Słowa kluczowe: strefa denna, usuwanie piasku, układ dynamiczny, punkt równowagi, stabilność równowagi, katastrofa elementarna, diagnostyka, skuteczność.

Corresponding author: R.M. Mammadov, e-mail: *ramil.mammadov.mu@asoiu.edu.az*

Article contributed to the Editor: 23.02.2024. Approved for publication: 18.07.2024.

Introduction

Scientists often describe events using mathematical models. Indeed, when such models are highly successful, they are said to not only describe events, but also explain them. If the model can be reduced to a simple equation, it may even be called a law of nature. For three centuries, the superior method for constructing such models was differential calculus, invented by Newton and Leibniz. Newton himself expressed his laws of motion and gravity in terms of differential calculus, and Maxwell introduced this calculus into the theory of electromagnetism. Einstein's general theory of relativity also culminates in a set of differential equations. Many lesser-known examples could also be added to this list. However, differential equations as a descriptive language have an inherent limitation: they can only describe phenomena that occur smoothly and continuously. In mathematical terms, solutions to differential equations must be differentiable functions. Relatively few phenomena are so ordered and behave so "well". On the contrary, the world is full of unexpected transformations and unpredictable deviations that require non-differentiable functions (Dlab and Ringel, 1974; Jensen et al, 2000; Navidi, 2011).

Catastrophe theory is the invention of René Thom. The term "bifurcation" is used, like "catastrophe", to designate qualitative rearrangements of various systems when parameters change (Castrigiano and Hayes, 2004). A common example of a catastrophe, or bifurcation, is the behavior of any elastic structure under an increasing load, which suddenly and abruptly moves to another position, making the direction of the structure's bending impossible to predict. Catastrophe theory is a mathematical method for dealing with discontinuous and divergent phenomena and may provide a mathematical language for the most sciences (Zeeman, 1976).

The theory can be used as mathematical apparatus with exceptional effectiveness in situations where gradual changes in strength or motivation lead to abrupt changes in behavior. For this reason, this method is called the catastrophe theory. A large number of events in physics can now be recognized as examples of mathematical catastrophes. Ultimately, however, the most productive applications of the theory can be found in biology and sociology, where discontinuous and divergent phenomena are ubiquitous and where other mathematical approaches have already been found ineffective. Catastrophe theory can thus provide a mathematical language for the hitherto "imprecise" sciences.

Catastrophe theory indicates some common features of phenomena involving abrupt changes in the regime of various systems in response to smooth changes in external conditions: a combination of randomness and necessity, determinism and unpredictability, and the ability to choose from several solutions near the bifurcation point (Luo et al., 1997; Gardiner, 2009). This approach, using the proposed apparatus, can be applied to assessing qualitative changes in sand conditions during hydrocarbon production.

Methodology

Timely and correct diagnosis of the onset of damage in the bottomhole zone is crucial. It allows for consideration of potential complications when choosing modes of further operation, planning necessary preventive measures, etc. At the same time, in the initial period of damage, its influence on the operation of wells is insignificant and cannot be detected using conventional research methods. As a rule, only after a certain period, when the process of sand removal progresses and quite significantly affects the operation of the well, is it concluded that complications may arise, making prevention must more difficult (Blanchard et al., 2006).

A technique for early diagnosis of the onset of near-wellbore zone distortion can be proposed based on the following considerations. The onset of damage corresponds to the beginning of a qualitatively new state of the "reservoir-well" system. As show by the results of studies conducted earlier, the appearance of sand in the product leads to instability of the operating mode and supply of the well, which persists for a considerable time. Diagnosing this qualitative shift and the change in the well's operating mode based on fluctuations in its productivity would allow us to conclude that the damage of the bottomhole zone has begun.

This problem can be addressed using catastrophe theory (Mirzajanzade and Stepanova, 1977; Mirzajanzade, 1986). The main principles of this theory are presented below.

A dynamic system, i.e. a process that continuously changes over time, is considered. The behavior of the system is modeled by specifying, in n-dimensional space, a trajectory point $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))$, representing the position of the system at time t. Obviously, if $x(t)$ remains in the same position for all t, then the system is in equilibrium with respect to the values x_i.

The evolution of the system can also be specified using a system of differential equations. In this case, the equilibrium position of the system corresponds to equality to zero for all $t'\dot{x}[t]$.

To describe a dynamic system in coordinate space $(x_1, x_2, x_3, \ldots, x_n)$, a certain smooth function V is introduced, such that the equilibrium points of the system coincide with the critical points (minimums) of V, i.e. with the points at which:

$$\frac{dV}{dx_1} = \frac{dV}{dx_2} = \ldots \frac{dV}{dx_n} = 0 \qquad (1)$$

Far from equilibrium positions, the function $V(x(t))$ decreases with increasing t.

The given model corresponds to dynamic systems with unchanged properties (Mirzajanzadeh and Shahverdiyev, 1997). In a more general case, the system, and, accordingly, the function V can depend on a number of parameters $C_1, C_2, ..., C_k$. In reality, this corresponds to cases when it is possible to physically control the system and change its properties. Obviously, the equilibrium position of such a parameterized system will change with changes in C. It may happen that as the parameters change, the equilibrium will become less and less stable, and finally, at certain values, it will become unstable or even unfeasible. In other words, at some values, there may be one minimum of the function V (stable equilibrium), while at others C there may be several minima (unstable equilibrium) (Ivakhnenko and Muller, 1984; Kauffman, 1993). In the latter case, it becomes possible to choose between different equilibrium states, and the behavior of the parameterized system will undergo a sudden "jump" from one equilibrium state to another.

Let us now formulate the problem in general terms. Let V be any smooth function of variables $(x_1, x_2, x_3, \dots, x_n)$ containing k parameters $(C_1, C_2, ..., C_k)$. In the control space $C = (C_1, C_2, ..., C_k)$, we define a set of catastrophes k for which V_c, as a function $x = (x_1, x_2, x_3, \dots, x_n)$, has several merging points. When $n = 1$, this means that k it is a set of points C such that V_c' and V_c'' simultaneously take a zero value in some x. When $n > 1$, k is the set of C such that all partial derivatives $\partial V_c/\partial x_1, \partial V_c/\partial x_2, ..., \partial V_c/\partial x_n$, as well as the determinant consisting of second derivatives

$$\begin{vmatrix} \dfrac{\partial^2 V_c}{\partial x_1 \partial x_2}, \dots, \dfrac{\partial^2 V_c}{\partial x_1 \partial x_n} \\ \dots\dots\dots\dots\dots\dots \\ \dfrac{\partial^2 V_c}{\partial x_n \partial x_1}, \dots, \dfrac{\partial^2 V_c}{\partial x_n \partial x_n} \end{vmatrix},$$

simultaneously vanish for some values $(x_1, x_2, x_3, \dots, x_n)$. From here we can obtain an equation for k, containing the parameters $(C_1, C_2, ..., C_k)$.

As the French mathematician Thom showed, for a widespread type of dynamical systems (evaluated by smooth functions containing no more than four parameters for any number of variables), in which abrupt changes in the output are observed, it is possible to give a geometric description of the paths along which such changes occur. He classifies these paths into 7 types, which are called elementary catastrophes (Thom, 1989; Arnold, 1992; Lorenz, 1993).

In accordance with the above, the change in well productivity (cumulative values) can be considered as a growth curve with saturation and described by a differential equation of the following form:

$$\dot{x} = ax^2 + bx + c \tag{2}$$

Where x is taken equal to $Q(t)$.

The potential function for equation (1) has the form:

$$V(x) = \frac{1}{3}ax^3 + \frac{1}{2}bx^2 + cx \tag{3}$$

And the critical region is located according to the following conditions:

$$\frac{dV}{dx} = 0, \ \frac{d^2V}{dx^2} = 0 \tag{4}$$

or

$$\begin{cases} ax^2 + bx + c = 0 \\ 2ax + b = 0 \end{cases} \tag{5}$$

Excluding x from (4) we obtain an expression for the critical region (determinant) $b^2 - 4ac = 0$. If the combination of control parameters a, b, c ensures that the condition $D < 0$ is met, then the system will be in an unstable position. Otherwise, if $D \succ 0$, the system is in a stable equilibrium position.

Calculations are carried out according to the following scheme:

- **1st step**: The total number of points N and values $x_1 = Q_i$ $(i = 1, ..., N)$ is entered;
- **2nd step**: The $x_i = [(Q_{i+1} - Q_i)1/2]$ values are calculated. Here h is the step with which the values of $Q_i(t)$ change;
- **3rd step**: The initial number N_1 of the experimental point from which calculations begin with a change step h is specified;
- **4th step**: The calculation interval $m = N_1 + h - 1$ is determined;
- **5th step**: The values a, b, c are calculated, for which equation (2) is reduced to the form:

$$\begin{aligned} a\sum_{i=N_1}^{m} x_i^4 + b\sum_{i=N_1}^{m} x_i^3 + c\sum_{i=N_1}^{m} x_i^2 &= \sum_{i=N_1}^{m} x_i^2 \dot{x} \\ a\sum_{i=N_1}^{m} x_i^3 + b\sum_{i=N_1}^{m} x_i^2 + c\sum_{i=N_1}^{m} x_i &= \sum_{i=N_1}^{m} x_i \dot{x} \\ a\sum_{i=N_1}^{m} x_i^2 + b\sum_{i=N_1}^{m} x_i + c(m - n + 1) &= \sum_{i=N_1}^{m} \dot{x} \end{aligned} \tag{6}$$

- **6th step**: The values $4ac$ and b^2 are calculated.
- **7th step**: If $4ac \prec b^2$, then the calculation of $m_1 = m + 1$ interval increases. If $m \le N$, then the repeating of the 5th step is necessary, and if $m \ge N$, proceed to the 8th step;

If $4ac \succ b^2$, then the values a, b, c for $m_1 = m - 1$ are calculated. In this case, the assignment to the value and the transition to the 4th step is made.

- **8th step**: End of calculation.

Calculations are carried out until the sign of the determinant changes, which corresponds to a "jump" in the parameters of the potential function through the critical zone, indicating a qualitative change in the behavior of the system (Navidi, 2011).

Table 1. Geological and technical indicators of the studied wells
Tabela 1. Parametry geologiczne i techniczne analizowanych odwiertów

N of the well	Horizon	Screen	Bottom-hole	Average production rate before sand registration $Q \cdot 1.16 \cdot 10^{-5}$
		[m]	[m]	[m³/sec]
1	Under Kirmakian 5	2597–2595	2607	220
2	Under Kirmakian 5	2589–2586	2594	215
3	Under Kirmakian 5	2607–2600	2611	210
4	Upper VI Horizon	2108–2080	2140	65
Note: The numbers of wells in the table are conditional				

Results and discussion

Next, we consider the practical application of this approach to diagnosing the onset of damage of the bottomhole zone. It should be noted that this approach has already found application in solving some problems of drilling and oil production (Dake, 1983; Mikhaylov, 1996; Mirzajanzadeh et al., 1997).

The operating data of wells 1, 2, 3 (the numbers are conditional) in area "X" and well 4 in area "Y" were used. Table 1 shows the geological and technical characteristics of these wells.

Figures 1–4 show the dynamics of the productivity of these wells (average flow rates for 5 days) from the start of operation until the first shutdown associated with slug formation.

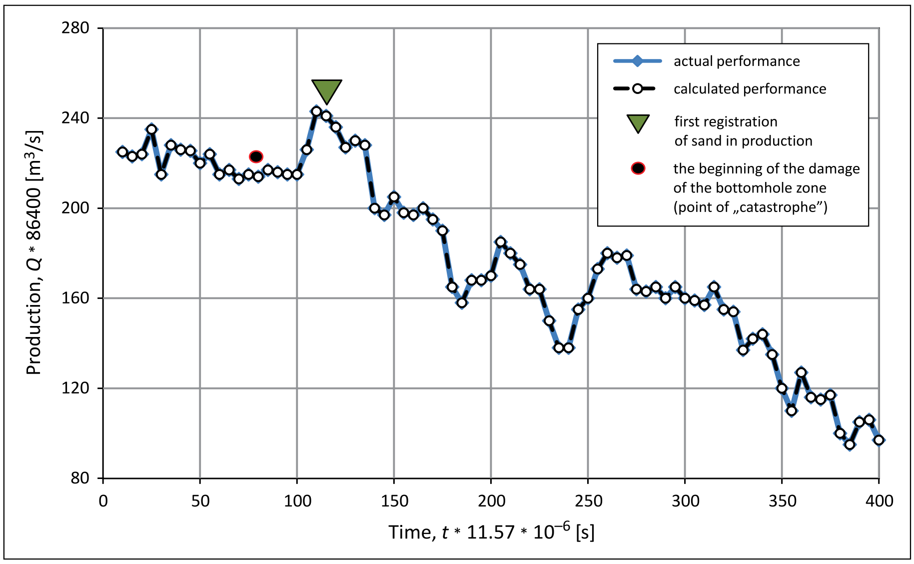

Figure 1. Dynamics of actual performance (liquid) for well 1 (Object X). The dependences of production indicators – Y axis ($Q \cdot 86\,400$ m³/s) on time – X axis ($t \cdot 11.57 \cdot 10^{-6}$ s)

Rysunek 1. Dynamika rzeczywistej wydajności (ciecz) dla odwiertu 1 (Obiekt X). Zależności wskaźników produkcji – oś Y ($Q \cdot 86\,400$ m³/s) od czasu – oś X ($t \cdot 11{,}57 \cdot 10^{-6}$ s)

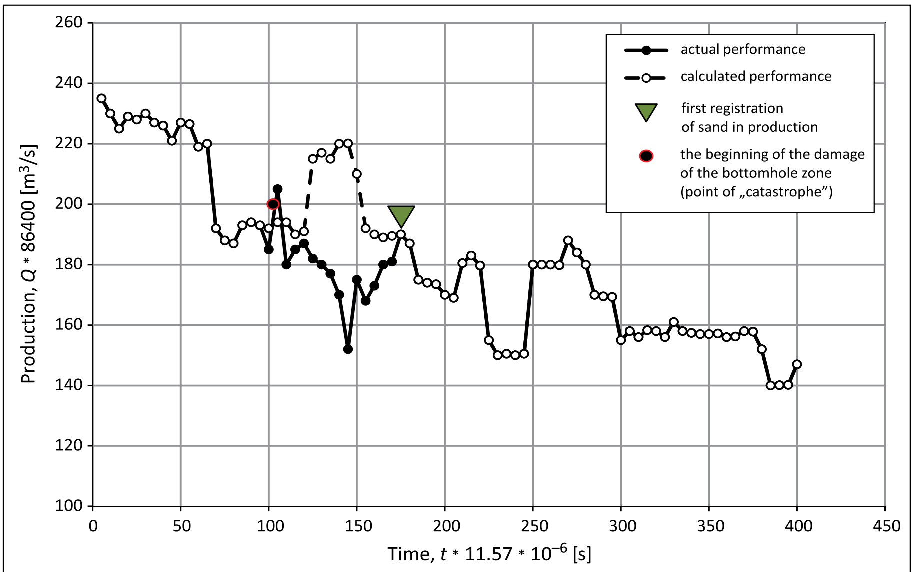

Figure 2. Dynamics of actual and calculated performances (liquid) for well 2 (Object X). The dependences of production indicators – Y axis ($Q \cdot 86\,400$ m³/s) on time – X axis ($t \cdot 11.57 \cdot 10^{-6}$ s)

Rysunek 2. Dynamika rzeczywistych i obliczonych wydajności (ciecz) dla odwiertu 2 (Obiekt X). Zależności wskaźników produkcji – oś Y ($Q \cdot 86\,400$ m³/s) od czasu – oś X ($t \cdot 11{,}57 \cdot 10^{-6}$ s)

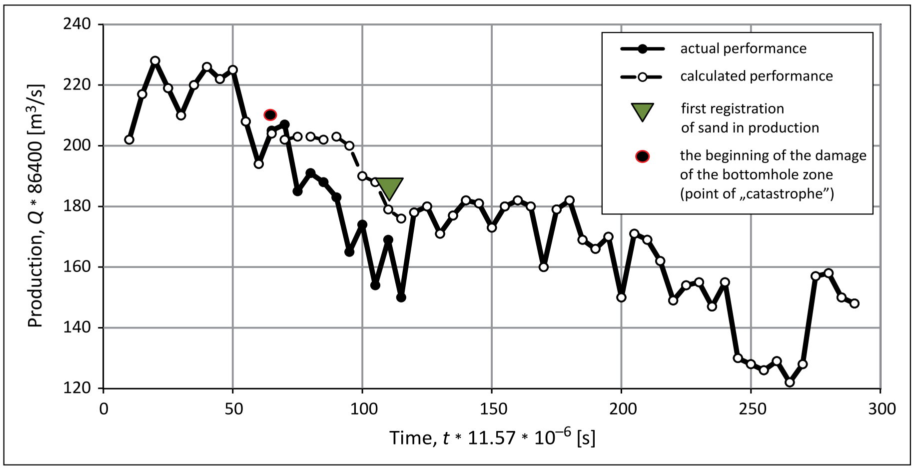

Figure 3. Dynamics of actual and calculated performances (liquid) for well 3 (Object X). The dependences of production indicators – Y axis ($Q \cdot 86\,400$ m^3/s) on time – X axis ($t \cdot 11.57 \cdot 10^{-6}$ s)

Rysunek 3. Dynamika rzeczywistych i obliczonych wydajności (ciecz) dla odwiertu 3 (Obiekt X). Zależności wskaźników produkcji – oś Y ($Q \cdot 86\,400$ m^3/s) od czasu – oś X ($t \cdot 11{,}57 \cdot 10^{-6}$ s)

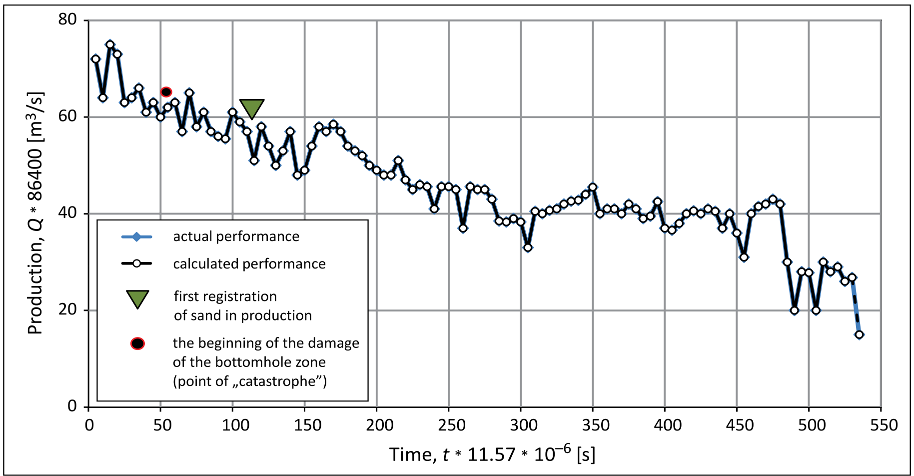

Figure 4. Dynamics of actual performance (liquid) for well 4 (Object Y). The dependences of production indicators – Y axis ($Q \cdot 86\,400$ m^3/s) on time – X axis ($t \cdot 11.57 \cdot 10^{-6}$ s)

Rysunek 4. Dynamika rzeczywistej wydajności (ciecz) dla odwiertu 4 (Obiekt Y). Zależności wskaźników produkcji – oś Y ($Q \cdot 86\,400$ m^3/s) od czasu – oś X ($t \cdot 11{,}57 \cdot 10^{-6}$ s)

The moments of the first registration of sand in well production are also indicated.

As calculations based on the theory of catastrophes have shown, in well 1, a "catastrophe" is observed already 35 days before the appearance of sand in the production and 320 days before the complete shutdown of the well. This corresponds to the beginning of distortion in the bottomhole zone. For well 2, the onset of damage in the bottomhole zone is diagnosed 70 days before the appearance of sand at the mouth and 325 days before stopping for repairs. In well 3, this moment is also recorded 60 days before registering sand in the production and 215 days before stopping to wash the sand plug. In well 4, this is observed 75 days before registering sand and 470 days before the first washing of the sand plug.

After the first "catastrophe" in the period before the complete damage and shutdown of the wells, several more qualitative changes in the productivity dynamics are observed in the wells under consideration. This fact is consistent with the mechanism of damage in the near-wellbore zone, during which there is an alternation of increase and decrease in the permeability of reservoir rocks, and, accordingly, the supply of wells.

The solution to the problem involved implementing an experiment consisting of the following elements: at the point that corresponds to the "catastrophe", the value of productivity changed with a certain step while simultaneously calculating the model coefficients a, b, c, and the determinant D. The performance value changed until the transition through the critical region was reached, indicated by a change in the sign of the determinant. Next, according to a similar scheme, subsequent performance values were selected (Mirzajanzade, 1986).

Conclusion

1. The foregoing demonstrates the feasibility of using the catastrophe theory to diagnose the onset of rock damage in the bottomhole zone.
2. This framework allows us to solve the problem of identifying the nature of changes in productivity, in which the noted qualitative changes ("catastrophes") do not occur.
3. Maintaining stability in the growth of cumulative production requires reduced extraction rates. For well 2, the decrease is from $202 \cdot 11.57 \cdot 10^{-6}$ to $175 \cdot 11.57 \cdot 10^{-6}$ m^3/s, that is, approximately 14%, and for well 3 – from $199 \cdot 11.57 \cdot 10^{-6}$ to $174 \cdot 11.57 \cdot 10^{-6}$ m^3/s, that is, about 14%. This

conclusion is consistent with the performance of several nearby wells from the same site.

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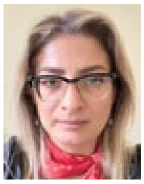
Ramil Mukhtar MAMMADOV, M.Sc.
Assistant at the Department of Oil and Gas Engineering
Azerbaijan State Oil and Industry University
34 Azadliq Ave., AZ1010 Baku, Azerbaijan
E-mail: *ramil.mammadov.mu@asoiu.edu.az*



Arzu Vagif SULTANOVA, M.Sc.
Scientific Researcher at the Scientific Research Institute “Geotechnological Problems of Oil, Gas & Chemistry”
Azerbaijan State Oil and Industry University
34 Azadliq Ave., AZ1010 Baku, Azerbaijan
E-mail: *arzu.sultanova.va@asoiu.edu.az*