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Differential equations expressing the thermal balance of metals and alloys

Równania różniczkowe wyrażające bilans cieplny metali i stopów

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ABSTRACT: One of the main directions of development in the oil and gas industry is the creation of new and efficient technological equipment. From 2020 to 2025, the increase in oil and gas production could be achieved through the correct selection of repair equipment and adherence to existing regulations for restoring wells that have been taken out of operation and returning them to service. Since the working parts of the equipment used in repair and restoration works in wells are constantly in contact with degradable objects, temperatures and thermal stresses increase due to friction at their contact points. High temperatures (1000–1200°C) create a stress-deformed state in the contact areas between tools and equipment. The stress-deformed conditions in the cutting zones lead to the formation of microcracks in the working part of the tool. Over time, these cracks grow, increasing temperature and thermal stresses, leading to tool wear, premature failure, and, in some cases, jamming. To maintain equipment and tools in working condition and ensure their periodic maintenance, the correct selection of milling parameters is necessary. The high productivity achieved during well drilling, repair, and restoration works depends on economic efficiency, durability, the choice of materials for cutting and milling tools that meet modern requirements, the compactness of construction, and the size and condition of the repaired well. One of the main factors ensuring the safe operation of equipment and tools in a well during repair work is maintaining the thermal-physical conditions of the moving parts of the tool at the required level. The thermal regime of the cutting tool largely depends on the physical and mechanical properties of the objects being destroyed: rock, metal, hardened cement, etc. When interacting with degradable objects, thermomechanical stresses affect the contacting surfaces of the tool, and a significant amount of heat is released at the working surface. As a result of extensive scientific and experimental research on the management of thermal-physical processes to enhance the stability of oil production, operational, and repair equipment, differential equations have been formulated to describe thermal processes. These equations incorporate coefficients of thermal conductivity, heat transfer, and cooling, which are fundamental to solving thermal problems.

Key words: temperature, thermomechanical stress, repair, well, plate, cylinder.

STRESZCZENIE: Jednym z głównych kierunków rozwoju przemysłu naftowego i gazowego jest projektowanie nowoczesnego i wydajnego sprzętu technologicznego. W latach 2020–2025 wzrost wydobycia ropy naftowej i gazu ziemnego można było osiągnąć dzięki prawidłowemu doborowi sprzętu naprawczego i przestrzeganiu obowiązujących przepisów dotyczących przywracania do eksploatacji odwiertów wyłączonych z użytkowania. Ponieważ elementy robocze urządzeń wykorzystywanych do naprawy i renowacji odwiertów są w ciągłym kontakcie z obiektami ulegającymi degradacji, następuje wzrost temperatury i naprężeń termicznych w wyniku tarcia w punktach ich styku. Wysokie temperatury (1000–1200°C) prowadzą do odkształceń naprężeniowych w miejscach styku narzędzi i sprzętu. Z kolei odkształcenia naprężeniowe w strefach skrawania prowadzą do powstawania mikropęknięć w części roboczej narzędzia. Z czasem pęknięcia te powiększają się, zwiększając temperaturę i naprężenia termiczne, co prowadzi do zużycia narzędzia, jego przedwczesnej awarii, a w niektórych przypadkach także zakleszczenia. Aby utrzymać sprzęt i narzędzia w stanie gotowości do pracy oraz zapewnić możliwość ich okresowej konserwacji, konieczny jest właściwy dobór parametrów frezowania. Wysoka wydajność osiągana podczas wiercenia, napraw i renowacji odwiertów zależy od efektywności ekonomicznej, trwałości, doboru materiałów na narzędzia skrawające i frezujące spełniające współczesne wymagania, a także od kompaktowości konstrukcji oraz wielkości i stanu naprawianego odwiertu. Jednym z kluczowych czynników zapewniających bezpieczną eksploatację sprzętu i narzędzi w odwiercie podczas prac naprawczych jest utrzymanie warunków termofizycznych ruchomych części narzędzia na wymaganym poziomie. Parametry termiczne narzędzia skrawającego zależą w dużej mierze od właściwości fizycznych i mechanicznych obrabianych obiektów: skał, metalu, stwardniałego cementu itp. Podczas pracy z obiektami ulegającymi degradacji, na powierzchniach styku narzędzia występują naprężenia termomechaniczne, a na powierzchni roboczej uwalniana jest znaczna ilość ciepła. W wyniku szeroko zakrojonych badań naukowych i eksperymentalnych nad regulacją procesów termofizycznych w celu zwiększenia stabilności produkcji ropy naftowej, sprzętu operacyjnego i naprawczego, opracowano równania różniczkowe opisujące procesy termiczne. Równania te wykorzystują współczynniki przewodnictwa cieplnego, wymiany ciepła i chłodzenia, które stanowią podstawę do rozwiązywania problemów termicznych.

Słowa kluczowe: temperatura, naprężenia termomechaniczne, naprawa, odwiert, płyta, cylinder.

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Introduction

One of the primary requirements for organizations engaged in scientific research and project development in the oil and gas industry is to enhance the productivity and durability of drilling and metal-cutting tools. Within the borehole, the forces acting on the working surfaces of equipment operating under high pressure, significant dynamic loads, and frictional forces are not evenly distributed. Consequently, a substantial portion of the energy expended on cutting and dispersing is dissipated to counteract internal and external forces that generate temperatures. The increase in stresses, friction velocities, and temperatures in moving parts adversely affects tool cooling and lubrication. These conditions, which arise during drilling and milling operations within the borehole, contribute to the formation and growth of microscopic cracks on the working surfaces of the tool, wear of cutting and dispersing elements, deterioration of operational conditions, chipping, and, ultimately, tool failure (Peteltskiy et al., 1971; Leontyev, 1979). During repair and restoration works in the wellbore, convective heat exchange and thermal radiation often occur simultaneously on the contact surfaces of cutting and breaking tools during technological operations. Heat conduction is determined by the distribution of thermal energy resulting from the mutual contact of individual particles of objects with different temperatures (Osipova, 1979; Mustafayev and Nasirov, 2022). Studying the patterns of heat transfer processes can help reduce, and sometimes partially eliminate, excessive temperatures on the contact surfaces of metals and other alloy compounds. Addressing thermal issues is critical for advancing theoretical knowledge in this domain. In the analytical theory of heat conduction, objects are considered as cohesive media. When the dimensions of the studied objects are significantly larger than molecular sizes and intermolecular distances, this approach is deemed more appropriate. Determining the coefficients of thermal conductivity, heat transfer, and heat exchange can contribute to reducing temperatures, temperature gradients, and thermomechanical stresses in the contact zones of cutting and breaking tools, eliminating stress-deformed states in the contact zone, and increasing the tool's resistance to temperatures (Ametistov et al., 1982; Klimenko and Zorin, 2001). Despite numerous research findings, technical literature rarely provides methods for determining the heat flux distribution coefficient during drilling and milling operations. Research indicates that compiling reports on heat transfer is challenging due to the lack of precise values for this coefficient. The heat flux distribution coefficient is better studied in brake systems under bending conditions (Mustafayev and Nasirov, 2022). However, since heat exchange during the heating and cooling of brake bearings in brake systems differs from heat exchange in the contact surfaces of cutting and breaking tools, the heat flux distribution coefficient determined for brakes cannot be directly applied.

Problem statement

To derive mathematical differential equations that incorporate temperature coefficients to reduce temperatures and thermal stresses on the contact surfaces of cutting and breaking tools when addressing thermal problems.

Discussion

Faradjiev and Dzhanakhmedov (1980) examined solutions to volumetric quasi-problems related to heat exchange in spherical axes subjected to heat flow on their outer surfaces and analyzed temperature differences in spherical bodies cooled by a liquid. In their study, the authors addressed the problem using linear equations for thermophysical variables. They considered various sources of heat generation, including those arising from supports, friction, cooling on the outer surface of the bit, convective cooling (contact with liquid), and conductive cooling (contact with air). However, they only partially accounted for the coefficients involved in thermal processes when determining temperatures in the contact zones of spherical bodies.

Faradzhev and Aliev (1982) presented a formula for differential equations defining a mathematical model of temperatures generated on the surfaces of objects with various shapes and sizes. While the authors derived differential equations for such bodies and systems of bodies, they did not consider heat transfer and cooling coefficients as primary factors in solving thermal problems, nor did they account for the physical and mechanical properties of metals and the rock with which the bodies interact.

Faradzhev et al. (1990) conducted a study on non-stationary temperatures and thermal stresses in the friction pair of the brake of drilling drawworks. Although the study aimed to reduce temperature stresses caused by friction in the friction zone, it was observed that thermal conductivity influenced heat dissipation. However, heat transfer and heat transfer coefficients were not accounted for in this article.

Faradzhev et al. (1991) discussed minimizing temperatures on the contact surfaces of a friction pair during oscillatory motion occurring in the braking system of a drill drawworks. The authors partially considered coefficients related to thermal issues.

In the works of Faradzhev et al. (1974, 1976, 1977, 1981a, 1981b, 1983) as well as Faradzhev and Rasulov (1975), the results of studies on thermal conductivity and heat transfer during drilling are of significant interest.

I the work of Faradzhev et al. (1977), the distribution of temperatures and thermal stresses on the contact surfaces of cylindrical objects was examined. However, the study did not account for thermal conductivity, heat transfer coefficients, and heat transfer as primary factors affecting temperature reduction.

Faradzhev et al. (1976) expanded the research carried out and solved isoperimetric variational problems to determine the optimal values of temperature stresses and the optimal shape of the working zone of the bit. The resulting shape and dimensions helped prevent crack formation on the surface of the tool during cooling. Although isoperimetric measures for reducing temperature stresses were identified, the cooling factor (Bi criterion) which affects cooling, was not considered.

Faradzhev et al. (1981b) theoretically determined temperature differences in the supports and contacting surfaces of the bit during drilling operations. The study revealed the negative impact of heat generated in the bit supports, leading to overheating of the bit and its working part. The authors proposed adjusting the bit operating parameters, such as drilling at low speeds, and, in some cases, replacing rolling bearings with plain bearings in the bit supports. However, they did not consider the heat generated by friction in the working part of the bit, which propagates throughout the entire volume of the bit, further heating the bit supports. The temperatures generated in the bit supports exceed normal temperatures, resulting in further heating of the tool and structural changes in its destructive elements, negatively affecting the drilling process.

In the work of Faradzhev et al. (1981a), the temperature-induced stresses in the supports of single-cored bits and their reduction are discussed. The authors managed to reduce the temperature loads on the working surface of the tool and on the supports by 10–20 times by adjusting the bit rotation speed. However, they did not account for heat transfer coefficients, heat transfer, and thermal conductivity.

Faradzhev et al. (1974) conducted a study of the thermal regimes affecting bit operation during drilling and analyzed the required amount of liquid for cooling heated sections of single-faceted bits. In addition to determining the amount of cooling liquid, the authors considered the quality of the liquid and the influence of the heat flow distribution coefficient on tool cooling. Prior to this study, heat flow distribution coefficients had not been considered in addressing temperature-related issues, as previous research primarily focused on drilling hard and brittle rocks. The authors proposed solutions that allowed for complete or partial bit cooling through circulation.

Faradzhev and Rasulov (1975) provided a more precise formulation of heating issues leading to the heating of single-cored bit supports, considering two heat sources (on the supports and in the working area). Although the authors accounted for heat exchange on the bit supports and within the working area, they did not consider the gap between the axis of symmetry of the bit and the cutter. Additionally, while they addressed temperature control issues in the drill axis supports and in the contact zone, they did not incorporate temperature coefficients into their analysis.

Faradzhev et al. (1983) investigated the thermal regime of the roller cutter support while considering variations in thermophysical parameters. Although the authors considered other coefficients included in the thermal regimes affecting the operation of the drill bit, they did not account for the cooling coefficient (Bi criterion).

Faradzhev et al. (1981c, 1981d) focused on determining the cooling coefficient in drilling and milling processes. Their study derived the cooling coefficient (Bi criterion) and its role in regulating temperatures affecting the cutting part of the tool. The research found that the cooling coefficient had only been partially considered in previous temperature-related studies. Furthermore, the dependence of temperatures and temperature gradients on the Bi criterion was established. It was determined that specific coefficients involved in heating processes in unsteady conditions could only be obtained through calculation. The authors also identified the dependence of the Bi coefficient on dimensionless quantities, proposing a method for determining the the cooling coefficient ($\bar{F}Bi$) in drilling and milling processes.

Methods of solving the problem

The problem is addressed through analytical investigation aimed at deriving a general equation for studying thermal problems in different media. The analytical study of heat conduction allows not only for the examination of temperature changes occurring in objects during interactions o over a certain period but also for determining a general equation that describes temperature dependence on interrelated factors.

The regulation and control of temperatures in the working zones of equipment are crucial for solving thermal problems. The configuration of objects subjected to thermal stresses, along with the formulation of mathematical equations for heat conduction, boundary, and initial conditions, enables the resolutions of issues related to thermal conductivity (Lykov, 1978; Osipova, 1979).

A key factor in preparing the working zones of oilfield equipment is the appropriate selection of materials and alloy compositions, considering the thermal stability of their structural elements.

Understanding the causes of temperature variations and developing strategies for their timely mitigation requires the development of rules that ensure heat exchange, taking into account certain physical and mathematical laws.

One of the primary tasks of this research is to identify the main factors influencing temperature distribution along the contacting surfaces and volumes of tools and equipment under different conditions, depending on their configuration and dimensions. Heat transfer equations include heat flow (Q), thermal resistance (R), absolute temperatures (T), temperature gradient (grad T), specific heat capacity (c), heat flux density (q), dimensionless temperatures (Θ), heat transfer coefficient (α), thermal conductivity coefficient (λ), molecular mass (μ), kinematic viscosity (δ), excess temperatures ($T\_ex.$), density (ρ), Stefan's constant – including such key factors as Boltzmann's constant (σ), time (t).

Studying the influence of these factors enables the determination of temperature transfer from one point in a body to another, or from one medium to another (for example, from solids to liquids and gases, and vice versa).

During technological processes involving metals, heat conduction, convective and conductive heat exchange, and heat radiation from surfaces often occur simultaneously.

Heat conduction arises from the distribution of thermal energy due to the mutual contact of individual particles with different temperatures.

Convective heat exchange typically results from the combined effects of heat transfer and heat conduction.

In energy systems, heat exchange occurs between different heat carriers that are separated from metal surfaces. The heat transfer process is accompanied by the presence of a wall that separates hot and cold surfaces. The analyzis of heat transfer mechanisms has the potential to contribute to the reduction or elimination of excessive temperatures in the contact areas of metals and other alloys. Solving practical problems in this domain is essential for advancing theoretical knowledge.

The primary reason for the distribution of thermal energy arising from the interactions between electromagnetic waves in individual particles of bodies with different thermal conductivity is energy exchange between micro-particles. Heat energy is generated through energy exchange in various mechanisms: chaotic motion of atoms in gases, propagation of elastic waves in liquids and dielectrics, energy exchange between free electrons in metals and other alloys, and energy transmission from free electrons to lattice atoms. In general, all physical phenomena involve changes in physical quantities that vary across space and time. Heat conduction and heat transfer processes can occur even when temperature distribution is non-uniform at different points in metals or alloy compounds. The heat conduction and heat transfer processes of solids or alloy compounds are accompanied by temperature variations both in spatial and temporal dimensions. Analytical studies of heat conduction allow to derive a special form of equation that determine the temperature change over a certain period of time during the interaction of objects, identifying the factors on which they depend. The temperature field is expressed by the following mathematical equation (Mustafayev and Khayrabadi, 2023):

$$T = f(x, y, z, t) \tag{1}$$

Equation (1) is a mathematical representation of the temperature field and a general expression of the equation, showing the dependence of the temperature field on time as temperatures move from one point of the object to another. The temperature field is divided into stationary and non-stationary fields, representing the collection of temperatures found at all points of the investigated medium for any time interval. In a stationary field, the temperature depends only on the following coordinates (Lykov, 1978; Ustyuzhanin et al., 1985).

$$T = f_1(x, y, z); \quad \frac{\partial T}{\partial t} = 0 \tag{2}$$

If the temperature function depends on two coordinates, the field becomes two-dimensional:

$$T = f_2(x, y, t); \quad \frac{\partial t}{\partial z} = 0 \tag{3}$$

If the temperature function depends on one coordinate, the field becomes one-dimensional:

$$T = f_3(x, t); \quad \frac{\partial T}{\partial y} = \frac{\partial T}{\partial z} = 0 \tag{4}$$

If points with the same temperature on the body are connected, an isothermal surface with uniform temperature (Figure 1) can be obtained. Since temperatures at different points of the body do not change simultaneously, isothermal surfaces do not intersect, either on the surface of the body or inside the body.

The temperature gradient is a unit vector directed in the direction normal to the isothermal surface (n_0) and numerically equal to the derivative of the temperature in that direction.

$\partial T/\partial n$ is the derivative of the temperature with respect to the normal n.

$$\text{grad } T = n_0 \frac{\partial T}{\partial n} \tag{5}$$

The derivative of the temperature $\partial T/\partial n$ is a scalar value that varies for different points on the isothermal surface. When the distance between isothermal surfaces Δn is small, the magnitude of the temperature gradient is greater.

$\partial T/\partial n$ is negative in the direction of decreasing temperature.

The temperature gradient (T) on the coordinate axes Ox, Oy, Oz is equal to:

$$\begin{cases} (\mathrm{grad}\ T)_x = \dfrac{\partial T}{\partial n}\cos(n,x) = \dfrac{\partial T}{\partial x} \\ (\mathrm{grad}\ T)_y = \dfrac{\partial T}{\partial n}\cos(n,y) = \dfrac{\partial T}{\partial y} \\ (\mathrm{grad}\ T)_z = \dfrac{\partial T}{\partial n}\cos(n,z) = \dfrac{\partial T}{\partial z} \end{cases} \quad (6)$$

According to the Fourier's hypothesis, the heat flow (dQ) passing through a unit area (dA) of an elementary isothermal surface in an elementary time interval (dt) is directly proportional to the temperature gradient $\partial T/\partial n$).

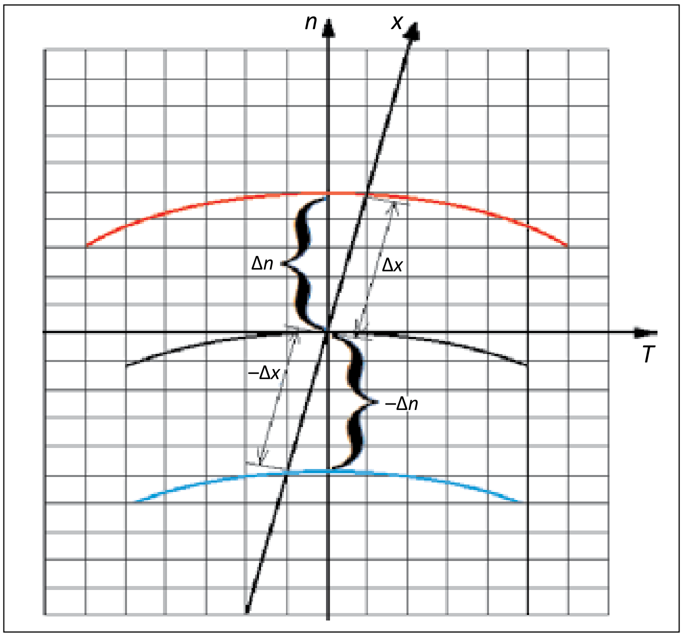

Figure 1. Isothermal surfaces
Rysunek 1. Powierzchnie izotermiczne

$$dQ_t = -n_0 \lambda \frac{\partial T}{\partial n} dA dt \quad (7)$$

The heat flux density passing through a unit area of an isothermal surface per unit time is determined by the following relationship:

$$q = \frac{dQ_t}{dA dt} \quad (8)$$

The heat flux density is determined by the following relationship:

$$q = -n_0 \lambda \frac{\partial T}{\partial n} \quad (9)$$

The heat flux density vector (q) is normal to the isothermal surface. The positive direction of the heat flux is concomitant with the direction of decreasing temperature, and heat is transferred from warmer parts of the body to cooler ones. The scalar value of the heat flux density vector is given by:

$$q = -\lambda \frac{\partial T}{\partial n} \quad (10)$$

Numerous experiments have confirmed the validity of Fourier's hypothesis. Thus, equation (9) is accepted as the fundamental mathematical expression of thermal conductivity, establishing that the heat flux density is directly proportional to the temperature gradient. If the temperature gradient varies at different points of the isothermal surface, then the total amount of heat passing through this surface per unit time is determined as follows:

$$Q = \int_A q dA = -\int_A \lambda \frac{\partial t}{\partial n} dA \quad (11)$$

dA – elementary isothermal surface in the i-th field.

The coefficient of thermal conductivity for various materials is typically determined experimentally. It is numerically defined as the ratio of the amount of heat passing through an isothermal surface to one temperature gradient.

$$\lambda = \frac{|q|}{|\mathrm{grad}\ T|} \quad (12)$$

Given the uneven distribution of temperatures in metals or composite alloys, both at the contacting surfaces and throughout the volume of the body, it is necessary to preliminarily determine the dependence of the heat transfer coefficient on temperature.

Numerous experiments show that, for some materials, a linear temperature dependence is used for a more accurate determination of the temperature dependence of thermal conductivity:

$$\lambda = \lambda_0 \left[1 + b(T - T_0)\right] \quad (13)$$

Where λ_0 is the thermal conductivity at the initial temperature (T_0), and b is a constant determined empirically.

In materials composed of various alloy compounds, the heat transfer coefficient sharply decreases. This is due to electron scattering caused by the heterogeneity of these alloy structures.

Unlike pure metals, the heat transfer coefficients of alloyed steels increase with increasing temperature.

Since Fourier's law applies to porous materials, the magnitude of the heat transfer coefficient must be considered. The amount of heat passing through the pores of molten compounds is equal to that passing through the pores of homogeneous bodies of various shapes and sizes.

The increase in temperature in fine-grained alloys is attributed to the increased thermal energy radiated from the pores into the mass.

Studying the physical phenomena in heat conduction requires determining various factors that characterize thermal while minimizing their number, and mathematically describing the differential equation necessary to establish the relationship between them. Let's consider an elementary mass parallel to the x, y, z axes and filling an elementary volume (dx, dy, dz) over an elementary time interval (dt) (see Figure 2) (Mustafayev et al., 1997).

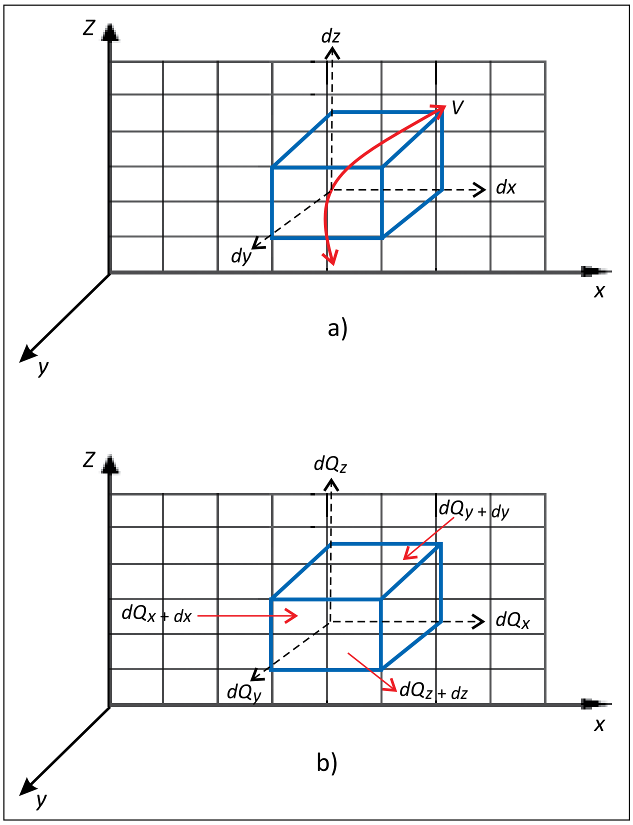

Figure 2. Scheme illustrating the heat transfer equation
Rysunek 2. Schemat ilustrujący równanie wymiany ciepła

To derive the differential equation, we make the following assumptions:

- The body is homogeneous and isotropic;
- Physical parameters are constant;
- Deformation of the considered volume due to temperature changes is negligible compared to the volume itself;
- Macroscopic particles of the body are stationary relative to each other.

In the general case, internal heat sources ($qv = f(x, y, z, t)$ within the body are uniformly distributed.

The amount of heat (dQ) supplied to the entire volume for an elementary time interval (dt) is equal to the sum of the changes in internal energy (dQ_1) and the heat removed from the elementary volume due to internal sources (dQ_2):

$$dQ_1 + dQ_2 = dQ \tag{14}$$

where:
dQ_1 – amount of heat supplied to the elementary volume for the elementary time interval (dt),
dQ_2 – amount of heat removed from the elementary volume (dV) due to internal sources for the time interval (dt),
dQ – change in internal energy of the body in the elementary volume (dV) for the time interval (dt).

To find the components of equation (14), consider an elementary parallelepiped with sides dx, dy, dz. The lateral faces of the parallelepiped should be parallel to the planes of the corresponding coordinate axes x, y, z. The amount of heat supplied along the faces of the parallelepiped in the direction of the x, y, and z axes for the time interval (dt) is denoted as dQ_x, dQ_y, dQ_z, respectively. If we need to determine the amount of heat emitted along the lines $dydz$ only in the x-axis direction for the elementary time interval (dt), then:

$$dQ_x = q_x\, dy\, dz\, dt \tag{15}$$

where: qx – projection of the heat flux density in the x-axis direction of the parallelepiped.

The difference in the quantity of heat transferred to or from the lateral surface of the elementary parallelepiped over the elementary time interval dt (dQ_{x1}) is given by:

$$dQ_{x_1} = dQ_x - dQ_{x+dx} \tag{16}$$

Since $qx + dx$ is continuous over the considered interval dx, we can expand it into a Taylor series:

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x}dx + \frac{\partial^2 q_x}{\partial x^2}\frac{\partial x^2}{2!} + \ldots \tag{17}$$

If we are satisfied with the first two terms of the Taylor series, then equation (16) can be written as follows:

$$dQ_{x_1} = -\frac{\partial q_x}{\partial x}dxdydzdt \tag{18}$$

Subsequently, the quantity of heat, denoted by dQ_1, can be determined through the implementation of a analogous principle for the amount of heat supplied to the volume of the elementary parallelepiped in the directions of the other two coordinate axes (ox and oz):

$$dQ_1 = -\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right)dxdydzdt \tag{19}$$

Next, we determine the quantity of the second component (dQ_2) in the equation (14).

The specific efficiency of internal heat sources (q_v) is regarded as the "volumetric heat transfer rate".

The value of heat density per unit time (dt) per unit volume (dV) of a body:

$$dQ_2 = q_v\, dVdt \tag{20}$$

The third component of equation (14), characterizing the change in internal energy (dQ), is determined as follows:

$$dQ = c\rho\frac{\partial T}{\partial t}dVdt \tag{21}$$

Substituting expressions (19), (20), and (21) into equation (14), we obtain:

$$\frac{\partial T}{\partial t} = -\frac{1}{c\rho}\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right) + \frac{q_V}{c\rho} \quad (22)$$

The heat amounts along the axes *ox*, *oy* and *oz* are determined by Fourier's law as follows:

$$q_x = -\lambda\frac{\partial T}{\partial x};\ q_y = -\lambda\frac{\partial T}{\partial y};\ q_z = -\lambda\frac{\partial T}{\partial z} \quad (23)$$

Using the expressions obtained in equation (22) for the projections of the heat flux vector onto the axes *ox*, *oy*, *oz*:

$$\frac{dT}{dt} = \frac{\lambda}{c\rho}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{q_v}{c\rho} \quad (24)$$

In equation (24):

$$\frac{\lambda}{c\rho} = a \quad (25)$$

Then we write:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \nabla^2 T \quad (26)$$

where:
a – thermal convection – coefficient [m^2/s],
$\nabla^2 T$ – Laplace operator in Cartesian coordinates.

If we consider (15) in (24), then the heat transfer equation can be expressed as follows:

$$\frac{\partial T}{\partial t} = a\nabla^2 T + \frac{q_v}{c\rho} \quad (27)$$

Equation (27) represents the differential equation of heat conduction, which characterizes the temperature change during heat conduction at any point within the body. The thermal conductivity coefficient (a) is a physical parameter used in solving non-stationary heat processes in metals and alloy compounds, characterizing the rate of temperature change. The heat transfer coefficient (α) characterizes the thermal conductivity of objects, while the coefficient (a) characterizes their thermal inertia properties.

From equation (24), it follows that the change in temperature over a given time ($\partial T/\partial t$), for any point within the object in space, is directly proportional to the value of the thermal conductivity coefficient (a). In other words, the greater the rate of temperature change at any point within the body, the higher the temperature transfer coefficient.

The thermal conductivity coefficient depends on the physical and mechanical properties of the object. Since liquids and gases exhibit high thermal inertia, they have low heat transfer coefficients, whereas metals and alloys, in contrast, have high heat transfer coefficients and therefore low thermal inertia. Since the differential equation of heat conduction is derived from the general laws of physics, it provides a general understanding of thermal energy transfer. Consequently, the differential equation of heat transfer encompasses all stages of the process.

Based on the conducted research, the following graphical dependencies have been established:

Figure 2 illustrates the relationship between temperature difference and the cooling coefficient (*Bi* criterion). As can be seen from the graph, as the cooling coefficient increases, the probability of a temperature difference decreases and gradually diminishes. In the intervals *Bi* 0.1–0.2, the process transitions to a steady-state regime.

Figure 3 illustrates the relationship between the temperature gradient and the cooling coefficient. As seen in the graph, as the cooling coefficient value increases (*after Bi* = 0.1), the temperature gradient transitions to a steady-state regime.

Figure 4 illustrates the relationship between temperature difference and the heat transfer coefficient. The graph shows that the temperature difference decreases as the heat transfer coefficient increases.

The obtained theoretical results provide valuable insights for determining heat transfer coefficients in differential equations, facilitating the modeling of thermal processes during the cooling and heating of contact surfaces between cylindrical tools and the plane of symmetry. Additionally, they aid in optimizing cooled surfaces cleaning from abrasives, stones, and other metallic particles.

If the temperatures within the body vary only along the body's thickness direction, then the equation governing heat exchange is expressed as follows:

$$\frac{\alpha\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \quad (28)$$

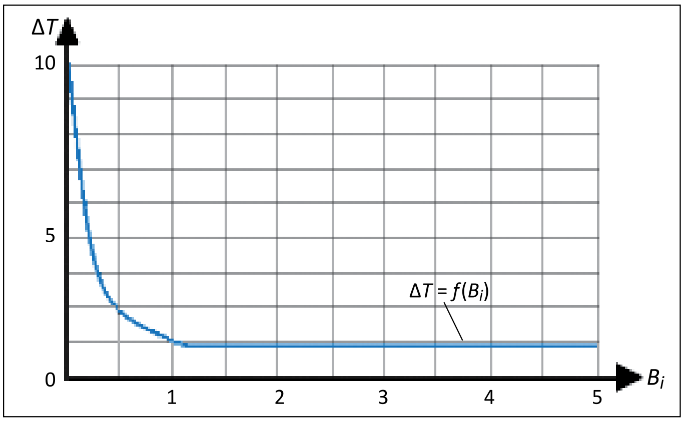

Figure 3. Graph showing the dependence of temperature difference (Δ*T*) on the cooling coefficient (*Bi* criterion)

Rysunek 3. Wykres przedstawiający zależność różnicy temperatur (Δ*T*) od współczynnika chłodzenia (kryterium *Bi*)

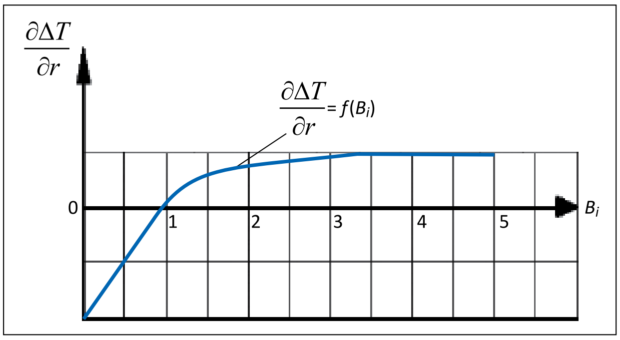

Figure 4. Graph illustrating the dependency of temperature gradient ($(\partial\Delta\overline{T})/\partial r$) on the cooling coefficient (*Bi* criterion)

Figure 4. Wykres ilustrujący zależność gradientu temperatury ($(\partial\Delta\overline{T})/\partial r$) od współczynnika chłodzenia (kryterium *Bi*)

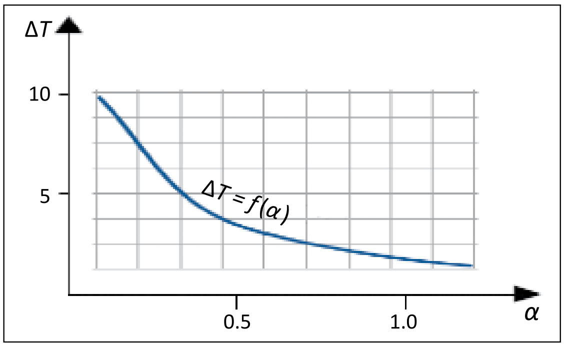

Figure 5. Graph depicting the relationship between temperature difference (ΔT) and the heat transfer coefficient (α)

Rysunek 5. Wykres przedstawiający zależność między różnicą temperatur (ΔT) a współczynnikiem przenikania ciepła (α)

Initial conditions (28):

$$T_t = T_0 = \text{const} \qquad (29)$$

The third type of boundary conditions involves the balance of two heat fluxes: heat transfer from the deeper layers of the cooled object to its surface

$$\left(q_{x=\delta} = -\lambda\left(\frac{\partial T}{\partial x}\right)_{x=\delta}\right)$$

and heat transferred from the surface to the coolant $q = \alpha(T_s - T_{env.})$:

$$-\lambda\left(\frac{\partial T}{\partial x}\right)_{x=\delta} = \alpha\left(T_s - T_{env.}\right) \qquad (30)$$

Since $x = 0$ according to the symmetry condition of temperature fields, then:

$$\left(\frac{\partial T}{\partial x}\right)_{x=0} = 0 \qquad (31)$$

Equations (28)–(31) are typically solved analytically by transforming them into dimensionless form:

$$\overline{T} = \sum_{n=1}^{\infty} \frac{2\sin\mu_n}{\mu_n + \sin\mu_n \cos\mu_n} \cos(\mu_n X)\exp(-\mu_n^2 Fo) \qquad (32)$$

Where: $\overline{T} = (T_s - T_{\text{env.}})/(T_{s0} - T_{\text{env.}})$ – dimensionless temperatures; μ_n – root of characteristic equations, (ctg $\mu_n = \mu_n)/Bi$); F_0 – Fourier number (dimensionless time), $F_0 = at/\delta^2$; *Bi* – *Bio*-number, $Bi = \alpha\delta/\lambda$. The *Bi*-number characterizes the ratio of heat transfer resistance due to thermal conduction from the center of a solid body to its surface ($R_\lambda = \delta/\lambda A$) to the heat transfer resistance due to thermal convection ($R_\lambda = 1/\alpha A$).

For heat-treated thin, long products, the *Bi* number approaches zero ($Bi \rightarrow 0$). The value of *Bi* is as assumed to be $Bi < 0.1$. According to equation (32), the report is maintained in the following order:

First, in the interval $0 - \pi/2$, the first root (μ_l – the first term of the series) of the equation ctg $\mu_n = \mu_n/Bi$ is found. Then it is necessary to reach it by shifting the interval μ_n relative to the previous one by π (up to the interval μ_{n-1}). Six members are sufficient to complete the series.

If the value of the Fourier number $F_0 > 0.3$, then the series can only be completed with the first term.

Nomograms $T = (F_0, Bi)$ are used to determine dimensionless temperatures in flat cylindrical and spherical bodies.

Temperature distribution curves over the plate thickness at various time intervals (coordinate systems ($\overline{T}$, X or T, X) are shown in Figure 5). The graph shows that the maximum temperatures are observed in the direction of the plate symmetry axis ($\overline{T}$).

If $F_0 > 0$ ($t > 0$) at any time interval, then the tangents drawn to the temperature distribution curves leave point *C*, located at a distance of $1/Bi$ from the side surfaces of the plate along the *X*-axis (Figure 5).

Expressing the boundary conditions of equation (30) as a dimensionless quantity allows for the derivation of the equation governing the temperature distribution on the surface of the cylinder:

$$(\partial\overline{T}/\partial X)_{X=1} = -Bi\overline{T}_s \qquad (33)$$

From Figure 5:

$$(\partial\overline{T}/\partial X)_{X=1} = -\text{tg}\varphi \qquad (34)$$

From (34):

$$\text{tg}\varphi = Bi\overline{T}_s \qquad (35)$$

Figure 5 shows that $\text{tg}\varphi = AB/AC$, where $AB = \overline{T}_s$, $AC = 1/Bi$.

For large values of B_i (practically $B_i > 100$) and if $x \gg \lambda/\delta$, then the distance $1/Bi$ approaches zero, i.e. $1/Bi \rightarrow 0$.

This indicates that it is possible to cool the surface of a heated body to the temperature of the liquid.

In such modes of heat transfer, the temperature change inside the body is determined only by the thermal resistance of heat conduction, and further increase of the heat transfer coefficient (α) does not accelerate the cooling process.

For small values of B_i, i.e. as B_i approaches zero, $Bi \rightarrow 0$, $AC = 1/Bi \rightarrow \infty$ approaches infinity, and the temperature does not change across the thickness of the plate (Figure 6).

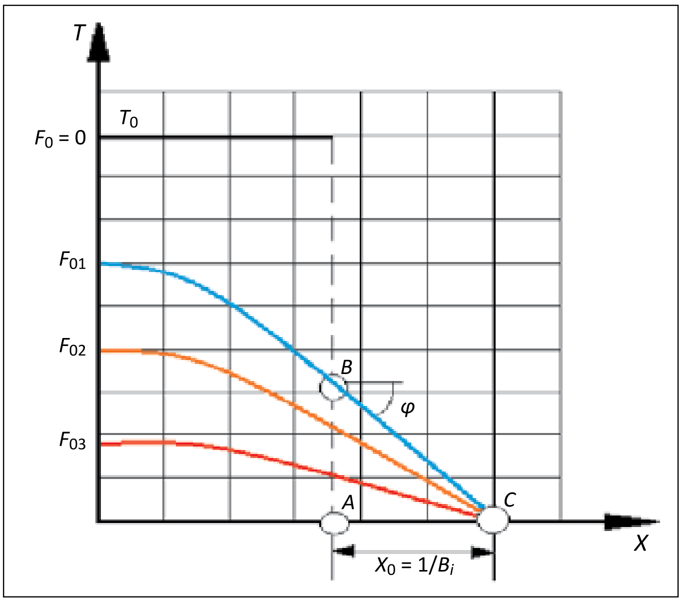

Figure 6. Diagram depicting the distribution of temperature on the surfaces of a cylinder

Rysunek 6. Wykres przedstawiający rozkład temperatury na powierzchni cylindra

Equation (32) can be used to calculate temperature fields in infinitely slender rectangular rods. Such bodies are considered as having abrasive intersections of infinitely slender rectangular plates. The temperatures at any point in these plates are assumed to be the derivatives of dimensionless temperature ($\overline{T}$), and these plates operate based on the intersections of these temperatures.

The results of theoretical research indicate that in both steady-state and non-steady-state regimes, the heat transfer of mono- and non-mono-materials, the transfer of heat from the surface to the surrounding environment, the distribution of temperatures at contact surfaces or along the volume of the body, and the determination and practical application of parameters governing these processes provide valuable insights for studying and applying heat transfer phenomena in various materials. The results obtained from theoretical research allow us to conclude the following:

According to the second law of thermodynamics, during heat transfer through conduction, the temperatures occurring throughout the volume of the body tend to decrease under the influence of the temperature gradient. Heat transfer in solid bodies and fluid mixtures occurs in accordance with the laws of heat conduction, and all the characteristics of the process are calculated based on the laws of heat exchange.

Regardless of whether it is in a liquid state or not, heat can be transferred in any substance, on its surface, inside it, and even in a vacuum.

Heat transfer in all substances occurs due to the vibration of micro-particles of energy. The velocity of molecules, atoms, electrons, and other micro-particles in substances subjected to heat varies proportionally to temperature. Micro-particles moving at high speeds and in mutual contact transfer their energy from hot parts of the body to cold parts (Ametistov et al., 1982).

Quasistatic temperature stresses are typically determined by thermoelastic potential displacement. This relationship is defined in the spatial coordinate system as follows:

$$\begin{cases} \sigma_{rr} = \dfrac{4G}{r}\dfrac{\partial \phi}{\partial r} \\ \sigma_{\theta\theta} = \sigma_{\varphi\varphi} = 2G\left(\dfrac{1}{r}\dfrac{\partial \phi}{\partial r} - \nabla \phi\right) \end{cases} \tag{36}$$

where:

σ_{rr} and $\sigma_{\varphi\varphi}$ – temperature stresses in the radial and tangential directions, respectively,

G – the second-order elastic modulus,

$\nabla = \dfrac{\partial r}{\partial r^2} + \dfrac{2}{r}\dfrac{\partial}{\partial r}$ – Laplace operator.

$\nabla \phi$ – thermoelastic potential displacement, which, depending on the temperature drop (ΔT), is given by:

$$\nabla \phi = \left(\frac{1+\mu}{1-\mu}\right)\beta \Delta T \tag{37}$$

where:

μ – Poisson's ratio,

β – coefficient of thermal expansion.

If (37) is differentiated with respect to time, then we obtain:

$$\frac{\partial \nabla \phi}{\partial t} = \left(\frac{1+\mu}{1-\mu}\right)\beta \frac{\partial T}{\partial t} = \left(\frac{1+\mu}{1-\mu}\right)\beta a \nabla \Delta T \tag{38}$$

Considering that the Laplace operator is time-independent, we can express (38) as follows:

$$\nabla \frac{\partial \phi}{\partial t} = \left(\frac{1+\mu}{1-\mu}\right)\beta a \nabla \Delta T \tag{39}$$

If we divide each side of (39) by '∇', then we obtain:

$$\frac{\partial \phi}{\partial t} = \left(\frac{1+\mu}{1-\mu}\right)\beta a \Delta T = k\Delta T \tag{40}$$

If we perform transformations in (40) using $x = r^2/4at$ as generalized parameters, we obtain:

$$\begin{cases} \dfrac{\partial \phi}{\partial t} = \dfrac{\partial \phi}{dx}\dfrac{\partial x}{\partial t} = -\dfrac{x}{t}\dfrac{d\phi}{dx} = k\Delta T \\ \dfrac{d\phi}{dx} = -kt\dfrac{\Delta T}{x} \end{cases} \tag{41}$$

Let's determine the second equation of (41) with respect to X:

$$\frac{d^2\phi}{dx^2} = \frac{d}{dx}\left(\frac{d\phi}{x}\right) = kt\frac{d}{dx}\left(\frac{\Delta T}{x}\right) = -kt\frac{1}{x}\left(\frac{d\Delta T}{dx} - \frac{\Delta T}{x}\right) \tag{42}$$

If we consider the Laplace operator in (36), we can express the temperature stresses as follows:

$$\begin{cases} \sigma_{rr} = -4G\dfrac{1}{r}\dfrac{\partial\phi}{\partial r} \\ \sigma_{\theta\theta} = \sigma_{\varphi\varphi} = -2G\left(\dfrac{1}{r}\dfrac{d\phi}{\partial r} + \dfrac{\partial^2\phi}{\partial^2 r}\right) \end{cases} \tag{43}$$

If we transform the radius derivative from a special compiler into a generalized variable x, we get:

$$\begin{cases} \dfrac{1}{r}\dfrac{\partial\phi}{\partial r} = \dfrac{1}{r}\dfrac{\partial x}{\partial r}\dfrac{d\phi}{dx} = \dfrac{1}{r}\dfrac{r}{2at}\dfrac{d\phi}{dx} = \dfrac{kt}{2at}\dfrac{\Delta T}{x} = -\dfrac{k}{2}\dfrac{\Delta T}{x} \\ \dfrac{\partial\phi^2}{\partial r^2} = \dfrac{d^2\phi}{dx^2}\left(\dfrac{\partial x}{dr}\right)^2 + \dfrac{d\phi}{dx}\dfrac{d^2x}{dr^2} = \dfrac{1}{at}\left(x\dfrac{d^2\phi}{dx^2} + \dfrac{1}{2}\dfrac{d\phi}{dx}\right) = \\ = -\dfrac{kt}{at}\left(\dfrac{d\Delta T}{dx} - \dfrac{\Delta T}{x} + \dfrac{1}{2}\dfrac{\Delta T}{x}\right) = -\dfrac{k}{a}\left(\dfrac{d\Delta T}{dx} - \dfrac{1}{2}\dfrac{\Delta T}{x}\right) \end{cases} \tag{44}$$

Substituting (36) into (35), the temperature stresses become:

$$\begin{cases} \sigma_{rr} = -4G\left(\dfrac{K}{2a}\cdot\dfrac{\Delta T}{x}\right) = 2G\dfrac{K}{a}\dfrac{\Delta T}{x} \\ \sigma_{\theta\theta} = \sigma_{\varphi\varphi} = -2G\left[-\dfrac{K}{2a}\cdot\dfrac{\Delta T}{x} - \dfrac{K}{a}\left(\dfrac{d\Delta T}{dx} - \dfrac{1}{2}\dfrac{\Delta T}{x}\right)\right] = \\ = 2G\dfrac{K}{a}\left[\dfrac{1}{2}\dfrac{\Delta T}{x} - \dfrac{d\Delta T}{dx} - \dfrac{1}{2}\dfrac{\Delta T}{dx}\right] = 2G\dfrac{K}{a}\dfrac{d\Delta T}{dx} \end{cases} \tag{45}$$

Using the value of k from (37), the temperature stresses and their differences can be written in a dimensionless form:

$$\begin{cases} \bar{\sigma}_{rr} = \dfrac{\sigma_{rr}}{2G\left(\dfrac{1+\mu}{1-\mu}\right)\beta T_{or}} = \dfrac{\Delta T}{x} \\ \sigma_{\theta\theta} = \bar{\sigma}_{\varphi\varphi} = \dfrac{\sigma_{\theta\theta}}{2G\left(\dfrac{1+\mu}{1-\mu}\right)\beta T_{or}} = \dfrac{d\Delta T}{dx} \\ \bar{\sigma}_{rr} - \bar{\sigma}_{\theta\theta} = \dfrac{\Delta T}{x} - \dfrac{d\Delta T}{dx} \end{cases} \tag{46}$$

Using formula (46), graphs have been constructed showing the dependence of temperature stresses due to friction on the dimensionless quantity x (for different values of the ratio x_1/x_0) (Figures 6 and 7).

As can be seen from Figure 6, there is a sharp decrease in the generated stress observed in the radial direction in the range $x \leq 1$ (for all values of the ratios x_1/x_0); after some time, the process stabilizes.

Analysis of the graph determined by formula (7) shows that when the ratio x_1/x_0 changes, the value of the stress difference also changes.

At small values of the ratio x_1/x_0, the stress difference is positive (Figure 7, curves 1 and 2), at values of $x_1/x_0 > 0.8$, the stress difference changes to negative.

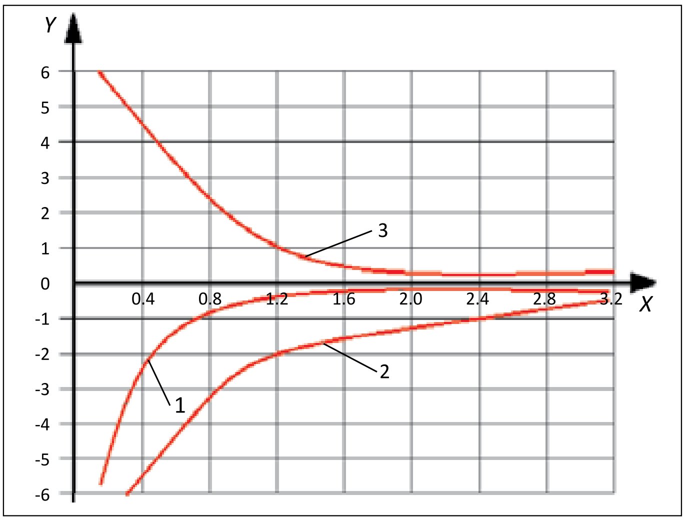

Figure 7. The graph showing the change of the radial stresses of the cylinder ($y = \sigma_{rr}$) depending on the parameter X (at different values of the ratio x_1/x_0) (on the graph: curve 1 – $x_1/x_0 = 0.2$; curve 2 – $x_1/x_0 = 0.5$; curve 3 – $x_1/x_0 = 1$)

Rysunek 7. Wykres przedstawiający zmianę naprężeń promieniowych cylindra ($y = \sigma_{rr}$) w zależności od parametru X (przy różnych wartościach stosunku x_1/x_0) (na wykresie: krzywa 1 – $x_1/x_0 = 0{,}2$; krzywa 2 – $x_1/x_0 = 0{,}5$; krzywa 3 – $x_1/x_0 = 1$)

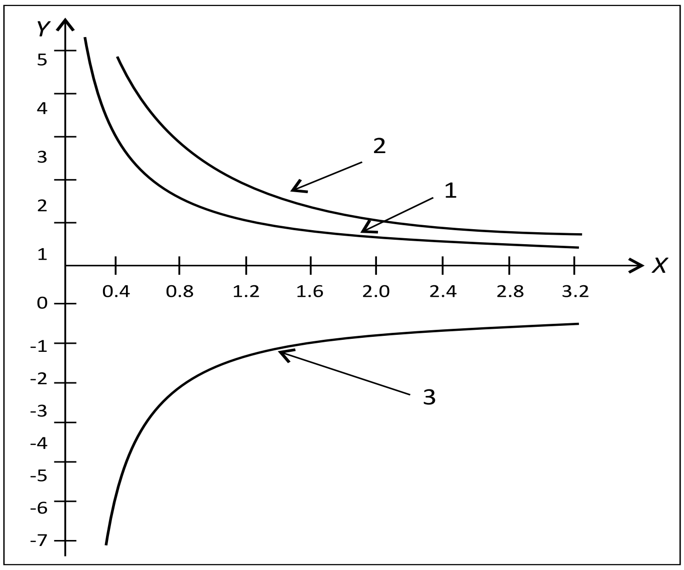

Figure 8. In the graph $y = \sigma_{rr} - \sigma_{\theta\theta}$ showing the change of the stress difference depending on the parameter X (at different values of the ratio x_1/x_0) (on the graph: curve 1 – $x_1/x_0 = 0.2$; curve 2 – $x_1/x_0 = 0.5$; curve 3 – $x_1/x_0 = 1$)

Rysunek 8. Wykres $y = \sigma_{rr} - \sigma_{\theta\theta}$ przedstawiający zmianę różnicy naprężeń w zależności od parametru X (przy różnych wartościach stosunku x_1/x_0) (na wykresie: krzywa 1 – $x_1/x_0 = 0{,}2$; krzywa 2 – $x_1/x_0 = 0{,}5$; krzywa 3 – $x_1/x_0 = 1$)

Therefore, since heat exchange exhibits a non-stationary nature, specific values should be determined by compiling reports using the methodology outlined above. It should be noted that the peculiarities of drilling and milling keep the tool in the destruction zone for a long time and do not allow to complete the process. It is necessary to periodically remove the tool from the destruction zone (for 1–2 minutes) while partially deactivating the heat source. During this time, the tool is continuously cooled by drilling fluid or other cooling components. Cooling is repeated until the process is complete to ensure that the tool can operate under normal conditions.

Executing the process in this sequence enables the partial deactivation of the heat source at the contacting surfaces of the tool for a controlled period. Regulation of temperatures and operating parameters, heat dissipation from the contacting surfaces out of the working zone, stable cooling, determination of the composition of the surface-activated cooling solution, and other technological means of repair and restoration allow to increase the productivity of the tool.

Conclusions

1. The temperature field has been determined, and general equations governing the dependence of the temperature field on time during the transition of temperatures from one point of the body to another have been established.
2. A significant reduction in the heat transfer coefficient has been observed when structural inhomogeneities arise in materials composed of various alloy compounds.
3. Equation (27), along with boundary conditions, expresses the complete mathematical formulation of a specific heat transfer problem.
4. The temperature stresses arising in the radial and tangential directions, respectively, and the differences between these stresses have been determined.

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