

Determination of hydraulic fracturing pressure during drilling by an analytical method

Wyznaczanie ciśnienia szczelinowania hydraulicznego podczas wiercenia metodą analityczną

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ABSTRACT: As well depth increases during drilling, the likelihood of various complications rises. These complications include drilling fluid loss, gas-oil-water influxes, wellbore stability issues, and pipe sticking. If appropriate preventive and corrective measures are not implemented, many of these complications may escalate into accidents. One such complication arising during well drilling is hydraulic fracturing of the formation. Hydraulic fracturing pressure is the pressure at which the integrity of the rock in the wellbore walls is compromised, leading to the formation of artificial fractures. Hydraulic fracturing during drilling is highly undesirable, as it results in the loss of drilling fluid into the surrounding formation. Hydraulic fracturing pressure depends on formation pressure, the natural fracturing of the rock, pore pressure, and the permeability of the rock and the fracturing fluid. As well depth increases, hydraulic fracturing pressure also increases and approaches formation pressure. Accurate determination of hydraulic fracturing pressure is therefore essential. The proposed method for determining a formation's hydraulic fracturing pressure is economically advantageous. This research paper presents a method for determining hydraulic fracturing pressure based on the energy condition of destruction, taking into account the mechanical properties of the rock and formation pressure.

Key words: well drilling, bottom-hole pressure, hydraulic fracturing pressure, hydrostatic head, free fall acceleration, energy condition of destruction, back pressure at the wellhead, stress state.

STRESZCZENIE: Wraz ze wzrostem głębokości odwiertu wzrasta prawdopodobieństwo wystąpienia różnych komplikacji, takich jak ubytki płuczki wiertniczej, dopływy gazu, ropy lub wody, problemy ze stabilnością odwiertu oraz zakleszczenie przewodu wiertniczego. Brak wdrożenia odpowiednich środków zapobiegawczych i naprawczych może skutkować przerodzeniem się wielu z tych komplikacji w wypadki. Jedną z takich trudności pojawiających się podczas wiercenia otworów jest szczelinowanie hydrauliczne skały. Ciśnienie szczelinowania hydraulicznego to ciśnienie, przy którym dochodzi do osłabienia ciągłości skał w ścianach odwiertu, co prowadzi do powstawania sztucznych szczelin. Szczelinowanie hydrauliczne podczas wiercenia jest zjawiskiem wysoce niepożądanym, ponieważ powoduje wyciek płynu wiertniczego do otaczającej formacji. Ciśnienie szczelinowania hydraulicznego zależy od ciśnienia złożowego, obecności naturalnych spękań w skałach, ciśnienia porowego, przepuszczalności skały oraz rodzaju płynu szczelinującego. Wraz ze wzrostem głębokości odwiertu, ciśnienie szczelinowania hydraulicznego również wzrasta i zbliża się do ciśnienia złożowego. Dlatego tak istotne jest precyzyjne określenie ciśnienia szczelinowania hydraulicznego. Proponowana metoda wyznaczania ciśnienia szczelinowania hydraulicznego formacji jest korzystna ekonomicznie. W niniejszej pracy zaprezentowano metodę wyznaczania ciśnienia szczelinowania hydraulicznego w oparciu o kryterium energetyczne zniszczenia, uwzględniającą właściwości mechaniczne skały oraz ciśnienie złożowe.

Słowa kluczowe: wiercenie otworu, ciśnienie przy dnie odwiertu, ciśnienie szczelinowania hydraulicznego, słup hydrostatyczny, przyspieszenie swobodnego spadania, kryterium energetyczne zniszczenia, przeciwcisnienie na głowicy odwiertu, stan naprężenia.

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Introduction

As the well depth increases, the geological conditions become more complex, increasing the risk of various complications. These complications make drilling difficult and require special measures for their prevention and mitigation. The most common complications include drilling fluid loss, gas-oil-water influxes, wellbore instability, and pipe sticking. If timely response measures are not taken, such situations may escalate into emergencies, posing a threat to operational safety.

Almost all complications and most accidents occur due to a mismatch between the actual pressure in the well and the required pressure. For example, if the pressure in the well is too low, the wellbore walls may collapse. If the pressure is too high, drilling fluid losses may occur. Therefore, it is essential to consider solutions to problems related to the pressures acting within the well, their influence on the drilling process, methods of determination, variations over time, and ways of regulation.

One of the complications that arise during drilling is hydraulic fracturing of the formation. Hydraulic fracturing pressure is the pressure at which the integrity of the formation in the wellbore walls is compromised, leading to the formation of artificial fractures. If the formation undergoes hydraulic fracturing during drilling, the drilling fluid penetrates the surrounding rock. Hydraulic fracturing pressure depends on the magnitude of the rock pressure P_r , the natural fracturing of the rock, P_f formation pressure, rock permeability, rheological properties, and the rate of fluid injection.

The hydraulic fracturing pressure can be determined as follows: the wellhead is sealed, and drilling fluid is injected into the drill pipe while continuously recording pressure and volume, generating a diagram as shown in Figure 1. In section OA, the formation resists the applied pressure. At point A, it begins to absorb fluid, resulting in a nonlinear pressure-volume relationship. At point B, the pressure exceeds the stresses in the

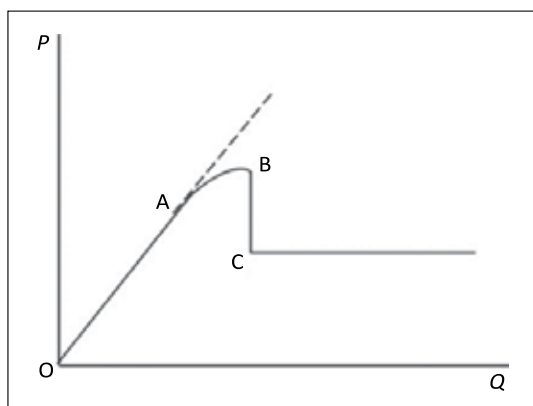


Figure 1. Dependence of pressure on the volume of injected fluid
Rysunek 1. Zależność ciśnienia od objętości wtryskiwanej cieczy

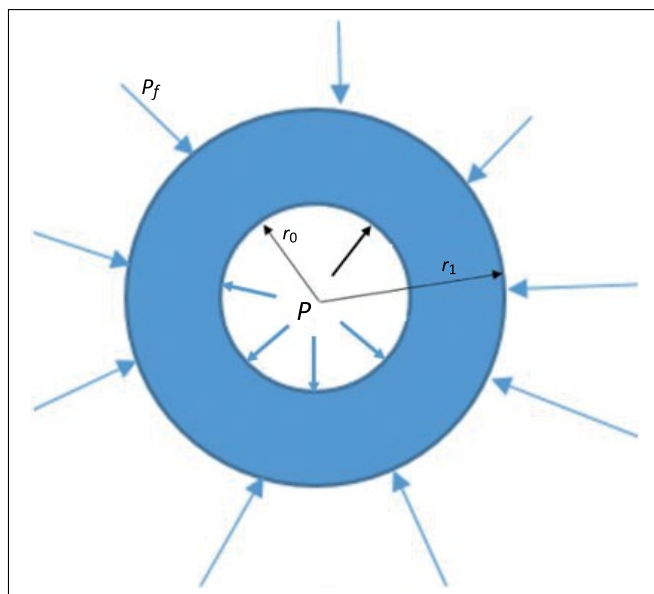


Figure 2. Load diagram

Rysunek 2. Schemat obciążenia

wellbore walls, leading to the formation of artificial fractures and a sharp pressure drop at the wellhead. This point represents the hydraulic fracturing pressure. The pressure at point C is referred to as the fracture propagation pressure.

Different authors have proposed formulas for determining the hydraulic fracturing pressure P_{hf} in the absence of actual data (Ludivine et al., 2012, 2014; Faergestad, 2016; Marbun et al., 2016):

$$P_{hf} = 0.083H + 0.66P_f \quad (1)$$

$$P_{hf} = 0.85(P_r - P_f) + P_f \quad (2)$$

$$P_{hf} = \frac{\mu}{1-\mu}(P_r - P_f) + P_f \quad (3)$$

$$P_{hf} = \frac{\mu}{1-\mu}P_r \quad (4)$$

where: μ is the Poisson's ratio, which typically ranges from 0.25–0.4 for dense clay, 0.33–0.4 for clay with sandstone interlayers, 0.1–0.2 for clay shales, 0.3–0.35 for sandstone, and 0.28–0.33 for limestone, P_r is the rock pressure and P_f is the formation pressure.

For most rocks, Poisson's ratio is as assumed to be 0.25.

Problem Solution

We will attempt to determine the hydraulic fracturing pressure P_{hf} using an analytical method.

Let's consider a wellbore as an infinite cylindrical body with an inner radius r_0 and an outer radius r_1 , subjected to internal wellbore pressure P and P_f formation pressure.

It is known that under axisymmetric conditions, the equilibrium equation for the near-wellbore zone takes the form (Faergestad, 2016):

$$\frac{d\sigma_r}{dr} + \frac{1}{r}(\sigma_r - \sigma_\varphi) = 0 \quad (5)$$

σ_r, σ_φ are the stresses that arise during the well development due to internal wellbore and reservoir pressure. The considered problem is a Lamé problem with boundary conditions when:

$$r = r_0 \quad \sigma_r = -P; \quad r = r_1 \quad \sigma_r = -P_f \quad (6)$$

Hooke's law for isotropic bodies in a cylindrical coordinate system is given by:

$$\begin{cases} \varepsilon_r = \frac{1}{E} [\sigma_r - \nu(\sigma_z + \sigma_\varphi)] \\ \varepsilon_\varphi = \frac{1}{E} [\sigma_\varphi - \nu(\sigma_r + \sigma_z)] \\ \varepsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_r + \sigma_\varphi)] \end{cases} \quad (7)$$

where:

E – Young's modulus,

ν – Poisson's ratio,

$\varepsilon_r, \varepsilon_\varphi, \varepsilon_z$ – relative deformations.

Since the transverse dimensions of the well are much smaller than its length, the well can be considered an infinitely long cylindrical body.

Therefore, it can be assumed that the arising deformations are plane, meaning that $\varepsilon_z = 0$. Then from the third equation (7):

$$\sigma_z = \nu(\sigma_r + \sigma_\varphi) \quad (8)$$

Substituting (8) into (7):

$$\begin{cases} \varepsilon_r = \frac{1}{E} (\alpha\sigma_r - \beta\sigma_\varphi) \\ \varepsilon_\varphi = \frac{1}{E} (\alpha\sigma_\varphi - \beta\sigma_r) \end{cases} \quad (9)$$

where: $\alpha = 1 - \nu^2$; $\beta = \nu + \nu^2$

After solving (9) for σ_r, σ_φ the following is obtained:

$$\begin{cases} \sigma_r = \frac{E}{\alpha^2 - \beta^2} (\alpha\varepsilon_r + \beta\varepsilon_\varphi) \\ \sigma_\varphi = \frac{E}{\alpha^2 - \beta^2} (\alpha\varepsilon_\varphi + \beta\varepsilon_r) \end{cases} \quad (10)$$

Let us denote the components of the displacement vector as u, v, w in the radial, tangential, and longitudinal directions, respectively. The problem is axisymmetric, therefore $v = 0$. The deformation is planar, therefore $w = 0$. As is well known, deformations are expressed through radial displacement as follows:

$$\begin{cases} \varepsilon_r = du/dr \\ \varepsilon_\varphi = u/r \end{cases} \quad (11)$$

Substituting (11) into (10) for the stresses we obtain:

$$\begin{cases} \sigma_r = \frac{E}{\alpha^2 - \beta^2} \left(\alpha \frac{du}{dr} + \beta \frac{u}{r} \right) \\ \sigma_\varphi = \frac{E}{\alpha^2 - \beta^2} \left(\alpha \frac{u}{r} + \beta \frac{du}{dr} \right) \end{cases} \quad (12)$$

Substituting (12) into (5) and after some transformations to determine the radial displacement, we obtain the following differential equation:

$$\frac{d^2u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} = 0 \quad (13)$$

It is known that the general solution of equation (13) has the form:

$$u = c_1 r + c_2 r^{-1} \quad (14)$$

where: c_1, c_2 – integration constants determined from boundary conditions (6).

The general solution of equation (5):

$$\begin{cases} \sigma_r = -\frac{1}{r_1^2 - r_0^2} \left[P_f r_1^2 \left(1 - \frac{r_0^2}{r^2} \right) + P r_0^2 \left(\frac{r_1^2}{r^2} - 1 \right) \right] \\ \sigma_\varphi = -\frac{1}{r_1^2 - r_0^2} \left[P_f r_1^2 \left(1 + \frac{r_0^2}{r^2} \right) - P r_0^2 \left(\frac{r_1^2}{r^2} + 1 \right) \right] \end{cases} \quad (15)$$

The energy failure criterion for isotropic bodies has the form (Orujov et al., 2016):

$$\sigma_r^2 + \sigma_\varphi^2 = 2\sigma_T^2 \quad (16)$$

where: σ_T – strength limit for isotropic rocks.

It is known that regardless of whether the cylindrical body is under external or internal pressure, plastic deformations always occur on the inner surface of the cylinder, where the stresses from (15) take the form when:

$$\begin{aligned} r = r_0; \quad \sigma_r = -P; \quad \sigma_\varphi = -\frac{2r_1^2}{r_1^2 - r_0^2} P_f + \frac{r_1^2 + r_0^2}{r_1^2 - r_0^2} P; \\ P = -2aP_f + bP \end{aligned} \quad (17)$$

Considering that the radius of the formation r_1 is much larger than the borehole radius r_0 , the values of a and b can be approximated as follows:

$$a = \frac{r_1^2}{r_1^2 - r_0^2} \approx 1, \quad b = \frac{r_1^2 + r_0^2}{r_1^2 - r_0^2} \approx 1 \quad (18)$$

If we consider (17) and (8) in (16), we obtain a 2nd order

equation with respect to P .

$$\begin{aligned} P^2 + (-2P_f + P)^2 &= 2\sigma_T^2 \\ P^2 - 2P_fP + 2P_f^2 - \sigma_T^2 &= 0 \end{aligned} \quad (19)$$

The obtained equation (19) has real solutions that uniquely determine the hydraulic fracturing pressure P_{hf} . The general solution to equation (19) is:

$$P_{hf} = P_f \pm \sqrt{\sigma_T^2 - P_f^2}$$

For real solutions, the following condition must be met:

$$\sigma_T^2 - P_f^2 \geq 0 \quad (20)$$

For any rock lying at a given depth, the strength limit is greater than the formation pressure.

Therefore, by knowing the formation pressure and the mechanical properties of the rock, we can determine the hydraulic fracturing pressure when drilling a well.

$$P_{hf} = P_f + \sqrt{\sigma_T^2 - P_f^2} \quad (21)$$

Conclusions

1. A formula has been derived to determine the hydraulic fracturing pressure that may lead to emergency situations during well drilling.
2. Based on this theory, a methodology can be developed to determine the hydraulic fracturing pressure as a function of the mechanical properties of the rock.

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