

Modeling, algorithmization, and solution of the optimal control problem for the ethylene production complex under conditions of incomplete information

Modelowanie, algorytmizacja i rozwiązanie problemu sterowania optymalnego dla kompleksu produkcyjnego etylenu w warunkach niepełnych informacji

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ABSTRACT: As a result of a comprehensive study of the technological units in the multidimensional and multi-connected gas fractionation subsystem of the EP-300 large-scale ethylene and ethane production complex, a generalized formulation of the optimization problem for a sequentially connected block of technological apparatus – including elements with fuzzy described states – was developed. Specifically, a mathematical formulation of the optimization problem for a technological complex for commercial ethylene production under conditions of incomplete information was formulated. Statistical models were constructed that link output flows with the quality indicators of the input flow, loads, and operating parameters. The necessity of constructing fuzzy regression models for the selective hydrogenation reactor of acetylene in an ethane-ethylene fraction and for the ethylene fractionation column is demonstrated due to the lack of information for their identification. For this purpose, the fuzzy regression method, the $\tilde{E} - D$ -estimation method, is proposed. As a result, a multidimensional nonlinear mathematical programming problem with a fuzzy objective function and some fuzzy constraints is obtained. To solve this problem, a transition to its clear deterministic analog is performed. For this purpose, the present article applies a universal method for solving mathematical programming problems with fuzzy parameters – namely, the method of (L-R)-representation of fuzzy numbers. The resulting multi-criteria nonlinear vector optimization problem concerning the operation of the technological complex for commercial ethylene production under conditions of incomplete information is solved using established vector optimization methods, particularly the methods of rigid and flexible priorities with two different weights. It is shown that solving the fuzzy problem using any of the proposed multi-criteria optimization methods yields in the production of commercial ethylene compared to solving the corresponding problem with a clear objective function.

Key words: technological complex, commercial ethylene, optimal control, hydrogenation reactor, vector optimization, mathematical model.

STRESZCZENIE: W wyniku kompleksowego badania jednostek technologicznych w wielowymiarowym i wieloetapowym podsystemie frakcjonowania gazu w wielkoskalowym kompleksie produkcji etylenu i etanu EP-300, opracowano uogólnione sformułowanie problemu optymalizacji dla sekwencyjnie połączonego bloku aparatury technologicznej – w tym elementów o stanach opisanych w sposób nieprecyzyjny. W szczególności sformulowano matematyczny opis problemu optymalizacji dla kompleksu technologicznego produkującego komercyjny etylen w warunkach niepełnych informacji. Skonstruowano modele statystyczne, które łączą strumienie wyjściowe ze wskaźnikami jakościowymi strumienia wejściowego, obciążeniami i parametrami eksploatacyjnymi. Wykazano konieczność skonstruowania modeli regresji rozmytej dla reaktora selektywnego uwodornienia acetyleny we frakcji etanowo-etylenowej oraz dla kolumny frakcjonowania etylenu ze względu na brak informacji do ich identyfikacji. W tym celu zaproponowano metodę regresji rozmytej, a konkretnie metodę estymacji $\tilde{E} - D$. W rezultacie otrzymano wielowymiarowy, nieliniowy problem programowania matematycznego z rozmytą funkcją celu i pewnymi rozmytymi ograniczeniami. W celu jego rozwiązania dokonano przejścia do jednoznacznego, deterministycznego odpowiednika. W tym celu w niniejszym artykule zastosowano uniwersalną metodę rozwiązywania problemów programowania matematycznego z parametrami rozmytymi, a mianowicie metodę reprezentacji (L-R) liczb rozmytych. Ostatecznie wielokryterialny problem wektorowej optymalizacji nieliniowej dotyczący funkcjonowania kompleksu technologicznego produkującego komercyjny etylen w warunkach niepełnej informacji rozwiązano przy użyciu znanych metod optymalizacji wektorowej, w szczególności metod sztywnych i elastycznych priorytetów z zastosowaniem

dwóch różnych wag. Wykazano, że rozwiązanie problemu rozmytego przy użyciu dowolnej z proponowanych metod optymalizacji wielokryterialnej prowadzi do poprawy wyników produkcji etylenu w porównaniu z rozwiązaniem odpowiadającego mu problemu z jednoznaczłą funkcją celu.

Słowa kluczowe: kompleks technologiczny, etylen komercyjny, sterowanie optymalne, reaktor uwodornienia, optymalizacja wektorowa, model matematyczny.

Introduction

Numerous scientific publications have addressed the optimization and modeling of technological apparatus for ethylene production. However, these works do not consider the control problem of ethylene production as a whole, and there are virtually no studies focused on the optimization of control for the technological complex for commercial ethylene production, upon which the purity and quality of the target product depend. To a significant extent, the quality of commercial ethylene depends on the choice of operating parameters for the hydrogenation reactor of acetylene compounds. The problem of real-time control of the hydrogenation reactor remains insufficiently developed, and there are no effective methods or models that account for the operational characteristics of this catalytic apparatus. The absence of specialized models and corresponding solution methods has necessitated the development of the approaches outlined in this article. Until now, optimization of technological processes in the low-temperature rectification apparatus used for ethylene production has been based on traditional models employing the equation of component-by-component material and heat balances, phase equilibrium, kinetic mass transfer dependencies of component installations, and associated optimization methods such as unconstrained optimization methods, linear programming, and dynamic programming. In most studies, process control within this subsystem has been reduced to solving the optimization problem for the static operating modes of the technological apparatus, with mathematical models represented as deterministic functions of operating parameters (Gubitoso, 2007; Sabri, 2014; Pisarenko et al., 2016; Samedov et al., 2019; Tovbin, 2023).

The technological complex for commercial ethylene production of the EP-300 plant, comprising sequentially connected apparatus, is a complex control object with incompletely defined states due to a lack of information in assessing the physico-chemical characteristics of the acetylene hydrogenation reactor. This is associated with the difficulty of determining the temperature profile of the reaction process and the proportion of the catalyst's active surface without direct control, as well as the inability to accurately assess the states of the selective hydrogenation process (Ibrahimov et al., 1991; Melikov, 2018, 2020; Melikov et al., 2023; Merdanov and Melikov, 2023; Melikov and Ibrahimli, 2024; Sardarova et al., 2024).

Therefore, due to the complex nature of the distribution of elementary reaction rates inside the reactor, the difficulty of monitoring raw material transformation within the apparatus, and frequent variations in raw material quality, the mathematical apparatus of fuzzy set theory is used to identify the model of the hydrogenation reactor when solving the optimal control problem of the complex for commercial ethylene production. Let us formulate a generalized statement of the optimal control problem for a complex for commercial ethylene production. As the optimization criterion, we will select the value of the target product output:

$$\tilde{F} = \sum_{j=1}^N \sum_{v=1}^V c_v^j \odot \tilde{y}_v^j \rightarrow \widetilde{max} \quad (1)$$

where: $\tilde{y}_v^j$ is the fuzzy variable representing the quantity of the v -th commercial product of the j -th installation, c_v^j is the price of the v -th commercial product of the j -th installation.

Equations describing the deterministic states of the apparatus link the loads and operating parameters to the output flows of the installations:

$$y_v^j = f^j(x_i^j, u_l^j), \forall i = \overline{1, M}, l = \overline{1, L}, j = \overline{1, P}, v = \overline{1, W} \quad (2)$$

where: x_i^j is the consumption (quality) of i -th raw material in the j -th apparatus, and u_l^j is the l -th control action of the j -th installation.

Models that account for the fuzzy nature of state assessments in reaction apparatus are described by equations with fuzzy parameters:

$$\begin{aligned} \tilde{y}_v^j &= \tilde{f}^j(x_i^j, \tilde{u}_{l,t-1}^j), \\ \forall i &= \overline{1, M}, l = \overline{1, L}, j = \overline{p+1, m}, v = \overline{w+1, h} \end{aligned} \quad (3)$$

where: $\tilde{u}_{l,t-1}^j$ is the fuzzy variable of the l -th control of the j -th reactor at time $t-1$.

Since the output flows of the m -th installation are fuzzy numbers, the equations of the subsequent $\overline{m+1, N}$ apparatus are also fuzzy:

$$\begin{aligned} \tilde{y}_v^j &= \tilde{f}^j(\tilde{x}_i^j, u_l^j), \\ \forall i &= \overline{1, M}, l = \overline{1, L}, j = \overline{m+1, mN}, v = \overline{h+1, V} \end{aligned} \quad (4)$$

where: $\tilde{x}_i^j$ is the fuzzy variable of the i -th input flow of the j -th installation.

Restrictions on the admissible values of operating parameters:

$$u_l^j \in U_l^j, \forall l = \overline{1, L}, j = \overline{1, N} \quad (5)$$

Equations that characterize the interdependencies of material flows for different interconnected apparatus are written in the form:

$$\sum_{i=1}^M x_i^j - \sum_{k=1}^K \sum_{v=1}^V H_v^{kj} y_v^j = 0, \forall j = \overline{1, N} \quad (6)$$

where: H_v^{kj} is a matrix characterizing the interconnections between the outputs of the k -th and inputs of the j -th production units. According to the regulations, additional constraints are introduced to meet the quality requirements for output streams:

$$\underline{y}_g^j \leq y_g^j(x_i^j, u_l^j) \leq \bar{y}_g^j, \forall j = \overline{1, N}, g = \overline{1, G}, l = \overline{1, L}, i = \overline{1, M} \quad (7)$$

where: $\underline{y}_g^j$ and $\bar{y}_g^j$ are the lower and upper constraints for the g -th output stream of the j -th installation.

Load range constraints:

$$\underline{x}_d^j \leq x_d^j \leq \bar{x}_d^j \quad (8)$$

where: $\underline{x}_d^j$ and $\bar{x}_d^j$ are the lower and upper limits on the d -th load of the j -th installation.

The quantitative values of the input and output flows of the apparatuses are subject to non-negativity requirements:

$$x_i^j \geq 0, \forall j = \overline{1, N}, i = \overline{1, M} \quad (9)$$

$$y_v^j \geq 0, \forall j = \overline{1, N}, v = \overline{1, V} \quad (10)$$

Thus, the optimal control problem of the studied complex (1)–(10), including a reaction apparatus operating under conditions of incomplete information, is formulated, where the models of the installations are described by (2)–(4), the ranges of the permissible control values u_l^j are (5), the equations for the connection of the apparatuses are (6), the requirements for the values of the loads are (8), (9), the sets of states y_v^j are (7), (10).

Mathematical statement of the optimal control problem

The optimal control problem for the complex for commercial ethylene production as a whole is written in the following form:

$$\begin{aligned} \tilde{F} &= \sum_{j=1}^N \sum_{v=1}^V c_v^j \odot \tilde{y}_v^j \rightarrow \widetilde{\max} \\ y_v^j &= f^j(x_i^j, u_l^j), \forall i = \overline{1, M}, l = \overline{1, L}, j = \overline{1, P}, v = \overline{1, W} \\ \tilde{y}_v^j &= \tilde{f}^j(x_i^j, \tilde{u}_{l,t-1}^j), \forall i = \overline{1, M}, l = \overline{1, L}, j = \overline{p+1, m}, v = \overline{w+1, h} \\ \tilde{y}_v^j &= \tilde{f}^j(\tilde{x}_i^j, \tilde{u}_{l,t-1}^j), \forall i = \overline{1, M}, l = \overline{1, L}, j = \overline{m+1, N}, v = \overline{h+1, V} \\ \sum_{i=1}^M x_i^j - \sum_{k=1}^K \sum_{v=1}^V H_v^{kj} y_v^j &= 0, \forall j = \overline{1, N} \\ \underline{y}_g^j \leq y_g^j(x_i^j, u_l^j) \leq \bar{y}_g^j, \forall j = \overline{1, N}, g = \overline{1, G}, l = \overline{1, L}, i = \overline{1, M} \\ \underline{x}_d^j \leq x_d^j \leq \bar{x}_d^j, \forall j = \overline{1, N}, g = \overline{1, G}, l = \overline{1, L}, i = \overline{1, M} \\ u_l^j &\in U_l^j, \forall l = \overline{1, L}, j = \overline{1, N} \\ x_i^j &\geq 0, \forall j = \overline{1, N}, i = \overline{1, M} \\ y_v^j &\geq 0, \forall j = \overline{1, N}, v = \overline{1, M} \end{aligned} \quad (11)$$

The optimization problem for the apparatus chain (11), including an element with fuzzy described states, applies to the complex for commercial ethylene production. This complex is one of the main blocks of the low-temperature rectification unit and consists of rectification columns (demethanization, deethanization, and ethylene fractionation) as well as a reactor for acetylene hydrogenation in the ethane-ethylene fraction (EEF). The reactor is a multidimensional and multi-connected apparatus with complex internal and external connections. Its operation is managed under conditions of information deficit due to limited understanding of the relationships between operating parameters and loads that vary over a wide range, the inability to control the temperature distribution within the reactor, and the degree catalyst deactivation. Consequently, there is a need to incorporate the informal experience of process personnel into reactor management. In addition, the variability of the magnitude and quality of the input load requires either continuous adaptation of models or the use of a mathematical framework capable of constructing models that remain valid under wide-ranging disturbance conditions. Therefore, optimization of the studied complex is reduced to solving the optimal control problem under conditions of incomplete information. The yield of commercial ethylene is considered as a criterion for the optimization problem of the complex:

$$\tilde{F} = C_{18} \odot \tilde{y}_{18} \rightarrow \widetilde{\max} \quad (12)$$

where: $\tilde{y}_{18}$ is a yield of commercial ethylene, and C_{18} is its unit price.

The mathematical dependencies necessary to describe the installations of the complex represent a system of constraints of the following form:

$$y_6 = f_1(x_1, x_2, x_3, x_4, x_5, u_1, u_2) \quad (13)$$

$$y_7 = f_2(x_1, x_2, x_3, x_4, x_5, u_1, u_2) \quad (14)$$

$$y_8 = f_3(x_1, x_2, x_3, x_4, x_5, u_1, u_2) \quad (15)$$

$$y_9 = f_4(x_1, x_2, x_3, x_4, x_5, u_1, u_2) \quad (16)$$

$$y_{10} = f_5(x_8, x_9, u_3, u_4) \quad (17)$$

$$y_{11} = 100 - y_{12} \quad (18)$$

$$y_{12} = f_6(x_8, x_9, u_3, u_4) \quad (19)$$

$$y_{13} = f_7(x_9, u_3, u_4) \quad (20)$$

$$y_{14} = f_8(x_8, x_9, u_3, u_4) \quad (21)$$

$$\tilde{y}_{15} = \tilde{f}_9(x_{10}, x_{11}, \tilde{u}_{5,t-1}, \tilde{u}_{6,t-1}) \quad (22)$$

$$\tilde{y}_{16} = \tilde{f}_{10}(x_{10}, x_{11}, \tilde{u}_{5,t-1}, \tilde{u}_{6,t-1}) \quad (23)$$

$$\tilde{y}_{18} = \tilde{f}_{11}(\tilde{x}_{16}, \tilde{x}_{17}, u_7, u_8) \quad (24)$$

$$\tilde{y}_{19} = \tilde{f}_{12}(\tilde{x}_{17}, u_7, u_8) \quad (25)$$

where: x_1, x_2, x_3, x_4 are the amounts of pyrogas entering the K-10 demethanization column from the compression subsystem, x_5 is the methane content in the feed to the K-10 column, y_6 and y_7 are, respectively, the ethylene content in methane and the methane flow rate from the top of column K-10, y_8 and x_8 are the amounts of the hydrocarbon mixture c_2 and higher,

respectively, in the bottom of column K-10 and the feed to the deethanization column K-11, y_9 and x_9 are the content of the sum of c_2 hydrocarbons, respectively, in the bottom of column K-10 and the feed of column K-11, y_{10} and y_{11} are, respectively, the amount of EEF and the ethylene content in the top product of column K-11, x_{10} and x_{11} are, respectively, the flow rate of EEF to reactor R-2 and the ethylene content in the EEF, y_{12} is the propylene content in the distillate of column K-11, y_{13} is the flow rate of mixture c_3 and higher from the bottom of column K-11, y_{14} is the content of ethylene and ethane in the bottom of column K-11, $\tilde{y}_{17}$ is a fuzzy variable describing the flow rate of EEF from reactor R-2, $\tilde{y}_{15}$ and $\tilde{y}_{16}$ are fuzzy variables describing the ethylene and acetylene content, respectively, in the EEF at the reactor outlet, $\tilde{x}_{16}$ and $\tilde{x}_{17}$ are fuzzy variables describing the acetylene content and the amount of EEF in the feed to columns K-12,13, $\tilde{y}_{18}$ and $\tilde{y}_{19}$ are fuzzy variables describing the amounts of ethylene and ethane, respectively, at the outlet of the K-12,13 ethylene fractionation column, u_1 and u_2 , u_3 and u_4 , u_7 and u_8 are the temperatures of the top and the bottom of columns K-10, K-11, and K-12,13 respectively, $\tilde{u}_{5,t-1}$ and $\tilde{u}_{6,t-1}$ are fuzzy variables describing the reactor temperature and the hydrogen-to-acetylene ratio in the EEF at time $t-1$.

Conditions of material balance and equations of technological apparatus connection:

$$y_7 + y_8 \leq \sum_{i=1}^4 x_i \quad (26)$$

$$y_{13} + y_{10} \leq x_8 \quad (27)$$

$$\tilde{y}_{18} + \tilde{y}_{19} \leq \tilde{x}_{17} \quad (28)$$

$$x_8 = y_8 \quad (29)$$

$$x_9 = y_9 \quad (30)$$

$$x_{10} = y_{10} \quad (31)$$

$$\tilde{x}_{16} = \tilde{y}_{16} \quad (32)$$

$$\tilde{x}_{17} = \tilde{y}_{17} \quad (33)$$

Ranges of change in the qualitative and quantitative characteristics of the output flows of the installations, as well as the requirements for non-negativity of the flow values for the apparatus:

$$y_6 \leq y_6 \leq \bar{y}_6 \quad (34)$$

$$y_{13} \leq y_{13} \leq \bar{y}_{13} \quad (35)$$

$$\tilde{y}_{15} \leq \tilde{y}_{15} \leq \tilde{\bar{y}}_{15} \quad (36)$$

$$\tilde{y}_{16} \leq \tilde{y}_{16} \leq \tilde{\bar{y}}_{16} \quad (37)$$

$$x_i \geq 0, \forall i = \overline{1,5} \quad (38)$$

$$y_v \geq 0, \forall v = \overline{6,14} \quad (39)$$

$$\tilde{y}_v \geq \tilde{0}, \forall v = \overline{15,19} \quad (40)$$

Intervals of control actions change:

$$u_1 \leq u_1 \leq \bar{u}_1 \quad (41)$$

$$u_2 \leq u_2 \leq \bar{u}_2 \quad (42)$$

$$u_3 \leq u_3 \leq \bar{u}_3 \quad (43)$$

$$u_4 \leq u_4 \leq \bar{u}_4 \quad (44)$$

$$\tilde{u}_{5,t-1} \leq \tilde{u}_{5,t-1} \leq \tilde{\bar{u}}_{5,t-1} \quad (45)$$

$$\tilde{u}_{6,t-1} \leq \tilde{u}_{6,t-1} \leq \tilde{\bar{u}}_{6,t-1} \quad (46)$$

$$u_7 \leq u_7 \leq \bar{u}_7 \quad (47)$$

$$u_8 \leq u_8 \leq \bar{u}_8 \quad (48)$$

The optimal control problem of the complex is finally written in the form:

$$\tilde{F} = C_{18} \odot \tilde{y}_{18} \rightarrow \widetilde{\max}$$

$$y_6 = f_1(x_1, x_2, x_3, x_4, x_5, u_1, u_2)$$

$$y_7 = f_2(x_1, x_2, x_3, x_4, x_5, u_1, u_2)$$

$$y_8 = f_3(x_1, x_2, x_3, x_4, x_5, u_1, u_2)$$

$$y_9 = f_4(x_1, x_2, x_3, x_4, x_5, u_1, u_2)$$

$$y_{10} = f_5(x_8, x_9, u_3, u_4)$$

$$y_{11} = 100 - y_{12}$$

$$y_{12} = f_6(x_8, x_9, u_3, u_4)$$

$$y_{13} = f_7(x_8, x_9, u_3, u_4)$$

$$y_{14} = f_8(x_8, x_9, u_3, u_4)$$

$$\tilde{y}_{15} = \tilde{f}_9(x_{10}, x_{11}, \tilde{u}_{5,t-1}, \tilde{u}_{6,t-1})$$

$$\tilde{y}_{16} = \tilde{f}_{10}(x_{10}, x_{11}, \tilde{u}_{5,t-1}, \tilde{u}_{6,t-1})$$

$$\tilde{y}_{18} = \tilde{f}_{11}(\tilde{x}_{16}, \tilde{x}_{17}, u_7, u_8)$$

$$\tilde{y}_{19} = \tilde{f}_{12}(\tilde{x}_{17}, u_7, u_8)$$

$$y_7 + y_8 \leq \sum_{i=1}^4 x_i$$

$$y_{13} + y_{10} \leq x_8$$

$$\tilde{y}_{18} + \tilde{y}_{19} \leq \tilde{x}_{17}$$

$$x_8 = y_8$$

$$x_9 = y_9$$

$$x_{10} = y_{10}$$

$$\tilde{x}_{16} = \tilde{y}_{16}$$

$$\tilde{x}_{17} = \tilde{y}_{17}$$

$$y_6 \leq y_6 \leq \bar{y}_6$$

$$y_{13} \leq y_{13} \leq \bar{y}_{13}$$

$$\tilde{y}_{15} \leq \tilde{y}_{15} \leq \tilde{\bar{y}}_{15}$$

$$\tilde{y}_{16} \leq \tilde{y}_{16} \leq \tilde{\bar{y}}_{16}$$

$$x_i \geq 0, \forall i = \overline{1,5}$$

$$y_v \geq 0, \forall v = \overline{6,14}$$

$$\tilde{y}_v \geq \tilde{0}, \forall v = \overline{15,19}$$

$$u_1 \leq u_1 \leq \bar{u}_1$$

$$u_2 \leq u_2 \leq \bar{u}_2$$

$$u_3 \leq u_3 \leq \bar{u}_3$$

$$u_4 \leq u_4 \leq \bar{u}_4$$

$$\tilde{u}_{5,t-1} \leq \tilde{u}_{5,t-1} \leq \tilde{\bar{u}}_{5,t-1}$$

$$\tilde{u}_{6,t-1} \leq \tilde{u}_{6,t-1} \leq \tilde{\bar{u}}_{6,t-1}$$

$$u_7 \leq u_7 \leq \bar{u}_7$$

$$u_8 \leq u_8 \leq \bar{u}_8$$

The resulting problem is a nonlinear optimization problem with fuzzy parameters.

Modeling and algorithmization of the optimal control problem

Let us proceed to model the apparatuses and algorithmize the solution to the formulated problem. During modeling,

each rectification column is considered as an object whose inputs are the feed flow rate, feed quality, and top and bottom temperatures. The outputs of the object are the quantitative and qualitative characteristics of the rectification column's output stream. The columns are of the same type. The column top and bottom temperatures are the controls, and the flow rate and feed analysis are the controlled disturbances. Linear and nonlinear approximations were used for model identification. The obtained results of column model identification are summarized in Table 1.

Finally, the rectification column models have the following form:

1. demethanization column K-10:

- nonlinear model for the ethylene content in methane at the top of the column:

$$y_6 = -0.04323x_1 - 0.00032x_1^2 - 0.10001x_2 + 0.00159x_2^2 + 0.04929x_3 + 0.0022x_3^2 - 0.00421x_4 + 0.00158x_4^2 - 0.01668x_5 + 0.00062x_5^2 + 0.31962u_1 + 0.00144u_1^2 - 0.31264u_2 + 0.02872u_2^2 + 20.52124$$

- linear model for the methane flow at the top of the column:

$$y_7 = 0.14469x_1 - 0.0156x_2 + 0.87061x_3 - 0.21026x_4 - 0.09507x_5 - 0.00067u_1 - 0.6129u_2 + 10.82499$$

- linear model for the flow rate mixture of hydrocarbons c_2 and higher from the column bottom:

$$y_8 = 0.90739x_1 + 0.79843x_2 + 0.05703x_3 + 0.20882x_4 + 0.03504x_5 - 0.01682u_1 + 0.13228u_2 + 17.54576$$

- linear model for the content of the c_2 -hydrocarbons sum in the bottom product:

$$y_9 = -0.27294x_1 + 0.08091x_2 + 1.18648x_3 + 0.17484x_4 + 0.18684x_5 - 0.10068u_1 + 0.73561u_2 + 71.56131$$

2. deethanization column K-11:

- linear model for the EEF flow rate at the top of the column:

$$y_{10} = 0.79683x_8 + 0.05245x_9 - 0.10091u_3 - 0.92595u_4 + 48.57524$$

- linear model for the propylene content in the column distillate:

$$y_{12} = 0.01452x_8 + 0.03425x_9 + 0.07563u_3 - 0.57923u_4 + 1.34842$$

- linear model for the flow rate of a hydrocarbons c_3 and higher mixture from the column bottom:

$$y_{13} = 0.27284x_8 - 0.03749x_9 + 0.66626u_4 - 37.65547$$

- linear model for ethylene and ethane content in the column bottom product:

$$y_{14} = -0.03145x_8 + 0.02557x_9 - 0.08649u_3 + 0.05606u_4 - 6.9556.$$

Due to the insufficient equipment with measuring instruments, which prevents full control over the physical and chemical transformations of raw materials in the reactor, difficulty

in assessing the catalyst state and the temperature profile of the reaction zone, as well as the complexity and number and relationships between the reactor's state variables, constructing a mathematical description for this apparatus using traditional methods and models is challenging. This raises the problem of constructing a reactor model using the knowledge of process personnel, formalized using the apparatus of fuzzy set theory. In this case, the reactor outputs are described by fuzzy variables, which in turn leads to the fuzzy description of the ethylene fractionation column. When constructing models for the optimal control of the ethylene production block, the dependencies describing the reactor and the ethylene fractionation column should be based on data collected during a passive experiment. The traditional method for modeling is regression analysis. However, when processes inputs and outputs are fuzzy variables connected by fuzzy relationships, traditional statistical inference is inapplicable. This necessitates a method that enables modeling the dependencies between several variables while accounting for their fuzzy nature. This method is fuzzy regression (Tanaka et al., 1982).

The fuzzy regression method employed was the $\tilde{E}$ -D-estimation method (Wang and Li, 1990), briefly described below. Let $X_0, X_1, \dots, X_n$ be fuzzy variables taking values in the set of all fuzzy numbers $F(R)$. Let's assume a linear relationship:

$$X_0 = \beta_1 X_1 + \dots + \beta_{n-1} X_{n-1} + \beta_n X_n \quad (50)$$

where $\beta_i, i = 1, n$ are unknown real coefficients, and

$$X_n(x) = \begin{cases} 1, & \text{if } x = 1 \\ 0, & \text{if } x \neq 1 \end{cases} \triangleq I_1$$

Let $\tilde{E} = \sum_{j=1}^m |E(x_{0j}^*) - E(\bar{x}_{0j}^*)|$, where $E(x_{0j}^*)$ and $E(\bar{x}_{0j}^*)$ denote the entropies of the j -th fuzzy observed value and the j -th fuzzy estimate of the value x_{0j} , where $\bar{x}_{0j}^* = \sum_{i=1}^n \beta_i \cdot x_{ij}^*$. Now consider the entropy of fuzzy sets $(E(X) = \int_R \mu_x(x) dx)$, where the membership function is also a fuzzy set. Let us consider fuzzy sets whose membership function is given in parametric form:

$$\mu_X = N(a, b) = e^{-\pi(x-a/b)^2} \quad (51)$$

$$\text{when: } b = 0, N(a, 0) = \begin{cases} 1, & \text{if } x = a \\ 0, & \text{if } x \neq a \end{cases}$$

The criterion for solving the problem of estimating the real coefficients $\beta_1, \beta_2, \dots, \beta_n$ is the condition for minimizing the entropy integral deviation of the estimated values of the dependent variable from the entropy of its observed values:

$$\sum_{j=1}^m |E(x_{0j}^*) - E(\hat{x}_{0j}^*)| = \min_{\beta_1, \dots, \beta_n} \sum_{j=1}^m |E(x_{0j}^*) - E(\bar{x}_{0j}^*)| \quad (52)$$

provided that:

$$\Lambda_{j=1}^m D(x_{0j}^*, \hat{x}_{0j}^*) \geq h \quad (53)$$

where: D is the degree of closeness between fuzzy numbers, and h is a given standard of closeness ($h \in (0, 1]$),

selected in accordance with the requirements of the problem: $\hat{X}_{0j}^* = \sum_{i=1}^m \beta_i^* x_{ij}^*, j = \overline{1, m}$.

The formulated problem (52)–(53) can be transformed. It is known that:

$$\sum_{j=1}^m |E(x_{0j}^*) - E(\bar{x}_{0j}^*)| = \sum_{j=1}^m |b_{0j} - \sum_{i=1}^m |\beta_i| \cdot b_{ij}| \triangleq J(\beta_1, \dots, \beta_n) \quad (54)$$

and that the degree of proximity:

$$D(x_{0j}^*, \bar{x}_{0j}^*) = \begin{cases} \bar{\mu} \left[\frac{a_{0j} - \sum_{i=1}^n \beta_i a_{ij}}{b_{0j} + \sum_{i=1}^n |\beta_i| b_{ij}} \right] & \text{if } \frac{a_{0j} - \sum_{i=1}^n \beta_i a_{ij}}{b_{0j} + \sum_{i=1}^n |\beta_i| b_{ij}} \in (-a, a) \\ 0 & \text{otherwise} \end{cases} \quad (55)$$

otherwise: $D(x_{0j}^*, \bar{x}_{0j}^*) = 0$

Using the last expression, the problem (52)–(53) is written as:

$$\min_{\beta_1, \dots, \beta_n} J(\beta_1, \dots, \beta_n) \quad (56)$$

under the following conditions:

$$\sum_{j=1}^n \beta_i a_{ij} + \sum_{j=1}^n |\beta_i| b_{ij} \bar{\mu}^{-1}(h) \leq a_{0j} - \bar{\mu}^{-1}(h) b_{0j}, \quad j = 1, \dots, m \quad (57)$$

$$-\sum_{j=1}^n |\beta_i| a_{ij} + \sum_{j=1}^n \beta_i b_{ij} \leq a_{0j} - a b_{0j} \quad (58)$$

$$\sum_{j=1}^n |\beta_i| a_{ij} + \sum_{j=1}^n \beta_i b_{ij} \geq a_{0j} - a b_{0j} \quad (59)$$

Problem (56)–(59) can be transformed into a series of linear programming problems. As a result, the estimates $\beta_1^*, \dots, \beta_n^*$ of the unknown real coefficients are determined.

When constructing the models, the reactor is considered as an object with the following inputs: the flow rate of the EEF (F_{EEF}^{in}), the ethylene content in the EEF ($Q_{C_2H_4}^{in}$), the hydrogen-to-acetylene ratio in the EEF ($Q_{H_2}^{in}/Q_{C_2H_2}^{in}$), and the temperature in the reactor (T_{EEF}^{out}). The outputs are the ethylene content ($Q_{C_2H_4}^{out}$) and acetylene content in the EEF ($Q_{C_2H_2}^{out}$). The control actions are hydrogen-to-acetylene ratio in the EEF and the EEF temperature. The flow rate of the EEF and its ethylene content are controlled disturbances. In this case, F_{EEF}^{in} and $Q_{C_2H_4}^{in}$ are crisp variables. However, the variables ($Q_{H_2}^{in}/Q_{C_2H_2}^{in}$) and T_{EEF}^{out} are fuzzy. It is clear that the dependent variables of the models are $Q_{C_2H_2}^{out}$ and $Q_{C_2H_4}^{out}$.

The parametric assignment of terms in accordance with (51) for constructing the reactor regression model is provided in Table 1. The structural diagram of the reactor as a modeling object is presented in Figure 1.

A linear model of the ethylene content in the EEF at the reactor outlet was obtained:

$$\tilde{y}_{15} = -0.42177x_{10} \oplus 0.8513x_{11} \oplus 1.14775 \oplus \tilde{u}_5 \oplus 2.62864 \oplus \tilde{u}_6 \oplus 16.1368$$

and a linear model of the acetylene content in the EEF at the reactor outlet:

$$\tilde{y}_{16} = -0.77469x_9 \oplus 1.54689x_{10} \oplus 0.08771 \oplus \tilde{u}_5 \oplus 6.96143 \oplus \tilde{u}_6 \oplus 95.11523$$

When modeling, the ethylene fractionation column is considered as an object whose inputs are: the flow rate of purified

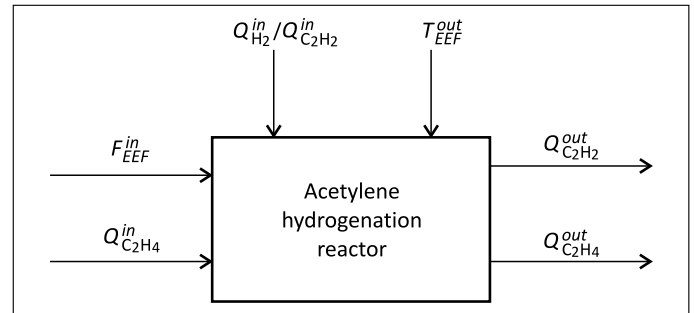


Figure 1. Structural diagram of the acetylene hydrogenation reactor as a modeling object

Rysunek 1. Schemat strukturalny reaktora uwodorniania acetyleny jako obiektu modelowania

Table 1. Parameters of mathematical models for acetylene hydrogenation reactor

Tabela 1. Parametry modeli matematycznych dla reaktora uwodorniania acetyleny

Variable name	Acetylene hydrogenation reactor			
	$Q_{H_2}^{in}/Q_{C_2H_2}^{in}$		T_{EEF}^{out}	
	a	b	a	b
Low	2.950	8.703	117	0.986
Medium	4.050	0.256	124	4.098
High	4.500	0.205	133	5.116

EEF coming from the acetylene hydrogenation reactor (F_{EEF}^{et}), the acetylene content in the purified EEF ($Q_{C_2H_2}^{et}$), as well as the top temperature (T_{top}^{et}) and bottom temperature (T_{cub}^{et}) of the column. The outputs of the object are the amounts of commercial ethylene ($F_{C_2H_4}^{out}$) and ethane ($F_{C_2H_6}^{out}$) produced in the ethylene fractionation column. The control actions are the top and bottom temperatures of the column. The purified EEF consumption and the acetylene micro-impurity content are considered controlled disturbances. The independent variables of the models T_{top}^{et} and T_{cub}^{et} are crisp. However, as shown above in the modeling of the reactor, the variables F_{EEF}^{et} and $Q_{C_2H_2}^{et}$ are fuzzy. Therefore, the dependent variables of the models $F_{C_2H_4}^{out}$ and $F_{C_2H_6}^{out}$ are also fuzzy.

The parametric assignment of terms for the ethylene fractionation column is provided in Table 2. The structural diagram of the ethylene fractionation column as a modeling object is shown in Figure 2.

A linear model for the ethane consumption from the column bottom has been constructed:

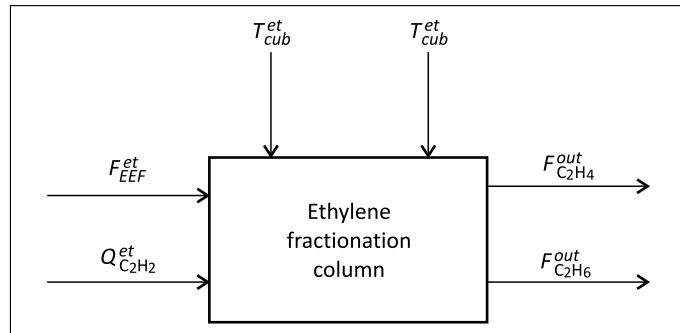
$$\tilde{y}_{19} = 0.19606\tilde{x}_{17} \oplus 4.22293u_7 \oplus 6.80709u_8 \oplus 85.91112$$

and a nonlinear model for the amount of commercial ethylene obtained in the column:

$$\tilde{y}_{18} = 14.98467 \oplus \tilde{x}_{17} \oplus 0.02088 \oplus \tilde{x}_{17}^2 \oplus 0.00789 \oplus \tilde{x}_{17} \oplus \tilde{x}_{16} \oplus 25.77907u_7 \oplus 0.34358 \oplus \tilde{x}_{17} \oplus u_7 \oplus 0.14156 \oplus \tilde{x}_{17} \oplus u_8 \oplus 0.12905 \oplus \tilde{x}_{16} \oplus u_8 \oplus 904.91$$

Table 2. Parameters of ethylene fractionation column models**Tabela 2.** Parametry modeli kolumn do frakcjonowania etylenu

Variable name Terms	Ethylene fractionation column			
	T_{EEF}^{out}		$Q_{C_2H_2}^{et}$	
	a	b	a	b
Low	36	4.098	$1 \cdot 10^{-4}$	$1.0241 \cdot 10^{-4}$
Medium	43	0.986	$5 \cdot 10^{-4}$	$1.7731 \cdot 10^{-4}$
High	52	4.098	$1 \cdot 10^{-4}$	$2.0471 \cdot 10^{-4}$

**Figure 1.** Structural diagram of the ethylene fractionation column**Rysunek 1.** Schemat strukturalny kolumny do frakcjonowania etylenu

Thus, a number of models have been constructed that describe both deterministic states of the apparatus (demethanization and deethanization columns) and fuzzy states of the apparatus (acetylene hydrogenation reactor and ethylene fractionation column) under conditions of incomplete information. Consequently, the optimization problem for the complex constitutes a mathematical programming problem with fuzzy parameters. Let us consider the algorithmization of the optimal control problem for the complex formulated as a mathematical programming problem with fuzzy parameters. This is a problem in which some of the constraints are fuzzy due to the fuzziness of the controls and the disturbances in the apparatus described by these constraints. To solve this problem, it is necessary to move on to a crisp analog for the given constraints. To achieve this, the method of solving mathematical programming problem with fuzzy parameters, the $(L-R)$ -representation of fuzzy numbers, is used below. Then the statement for the optimal control problem of the complex using extended arithmetic operations is written as:

$$\tilde{F} = C_{18} \odot \tilde{y}_{18} \rightarrow \widetilde{\max} \quad (60)$$

$$y_6 = a_{11}x_1 + a_{12}x_1^2 + a_{13}x_2 + a_{14}x_2^2 + a_{15}x_3 + a_{16}x_3^2 + a_{17}x_4 + a_{18}x_4^2 + a_{19}x_5 + a_{1,10}x_5^2 + b_{11}u_1 + b_{12}u_1^2 + b_{13}u_2 + b_{14}u_2^2 + a_{10} \quad (61)$$

$$y_7 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 + b_{21}u_1 + b_{22}u_2 + a_{20} \quad (62)$$

$$y_8 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 + b_{31}u_1 + b_{32}u_2 + a_{30} \quad (63)$$

$$y_9 = a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 + b_{41}u_1 + b_{42}u_2 + a_{40} \quad (64)$$

$$y_{10} = a_{51}x_8 + a_{52}x_9 + b_{51}u_3 + b_{52}u_4 + a_{50} \quad (65)$$

$$y_{11} = a_{61}x_8 + a_{62}x_9 + b_{61}u_3 + b_{62}u_4 + a_{60} \quad (66)$$

$$y_{12} = 100 - y_{11} \quad (67)$$

$$y_{13} = a_{71}x_8 + a_{72}x_9 + b_{71}u_3 + a_{70} \quad (68)$$

$$y_{14} = a_{81}x_8 + a_{82}x_9 + b_{81}u_3 + b_{82}u_4 + a_{80} \quad (69)$$

$$\tilde{y}_{15} = a_{91}x_{11} \oplus a_{92}x_{12} \oplus b_{91} \odot \tilde{u}_5 \oplus b_{92} \odot u_6 \oplus a_{90} \quad (70)$$

$$\tilde{y}_{16} = a_{10.1}x_{11} \oplus a_{10.2}x_{12} \oplus b_{10.1} \odot \tilde{u}_5 \oplus b_{10.2} \odot u_6 \oplus a_{10.0} \quad (71)$$

$$\tilde{y}_{18} = a_{11.1} \odot \tilde{x}_{17} \oplus a_{11.2} \odot x_{17}^2 \oplus a_{11.3} \odot \tilde{x}_{17} \odot \tilde{x}_{16} \oplus b_{11.1} \odot \tilde{x}_{17} \odot u_7 \oplus b_{11.2} \odot \tilde{x}_{17} \odot u_8 \oplus b_{11.3} u_8 \odot \tilde{x}_{16} \oplus b_{11.4} u_7 \oplus a_{11.0} \quad (72)$$

$$\tilde{y}_{19} = a_{12.1} \odot \tilde{x}_{17} \oplus b_{12.1} u_7 + b_{12.2} u_8 + a_{12.0} \quad (73)$$

$$\sum_{i=1}^4 x_i - y_7 - y_8 \geq S_1 \quad (74)$$

$$y_{10} + y_{13} - x_8 \leq S_2 \quad (75)$$

$$x_{10} - \tilde{y}_{17} \leq S_3 \quad (76)$$

$$x_{10} - \tilde{y}_{17} \geq 0 \quad (77)$$

$$\tilde{y}_{18} + \tilde{y}_{19} \leq \tilde{x}_{17} \quad (78)$$

$$\underline{y_6} \leq y_6 \leq \overline{y_6} \quad (79)$$

$$\underline{\tilde{y}_{14}} \leq \tilde{y}_{14} \leq \overline{\tilde{y}_{14}} \quad (80)$$

$$\underline{\tilde{y}_{15}} \leq \tilde{y}_{15} \leq \overline{\tilde{y}_{15}} \quad (81)$$

$$\underline{\tilde{y}_{16}} \leq \tilde{y}_{16} \leq \overline{\tilde{y}_{16}} \quad (82)$$

$$\underline{u_j} \leq u_j \leq \overline{u_j}, \quad \forall j = \overline{1.4} \quad (83)$$

$$\underline{u_7} \leq u_7 \leq \overline{u_7} \quad (84)$$

$$\underline{u_8} \leq u_8 \leq \overline{u_8} \quad (85)$$

$$x_i = y_v, \quad \forall i = v = \overline{8.11} \quad (86)$$

$$\tilde{x}_{16} = \tilde{y}_{16} \quad (87)$$

$$\tilde{x}_{17} = \tilde{y}_{17} \quad (88)$$

$$x_i \geq 0, \quad \forall i = \overline{1.5} \quad (89)$$

$$y_v \geq 0, \quad \forall v = \overline{7.13} \quad (90)$$

The constructed system (60)–(90) is formulated as a problem with fuzzy objective function and fuzzy constraints. The considered optimization problem for the acetylene hydrogenation reactor is solved as a mathematical programming problem with fuzzy parameters.

In the proposed approach to modeling the reactor state assessment, fuzzy numbers of the $(L-R)$ type (Dubois and Prade, 1979) are used. The membership function of such numbers has the form:

$$\mu_{\tilde{A}}(u) = \begin{cases} L\left(\frac{a-u}{\alpha}\right) & \text{for } u \leq a, \alpha > 0 \\ R\left(\frac{u-a}{\beta}\right) & \text{for } u \geq a, \beta > 0 \end{cases}$$

where: L is the left representation, R is the right representation, a is the medium value of the fuzzy number, α , and β are the

right and left fuzziness coefficients. L and R are increasing functions on the interval $[0, \infty)$. A fuzzy number of the $(L-R)$ type is written as:

$$\tilde{A} = (a, \alpha, \beta)_{LR}$$

Let us now transition from problem (60)–(90) to a system with crisp constraints and a crisp objective function. Let us write down the $(L-R)$ -representation for the equations describing the output flows of the catalytic apparatus:

$$\begin{aligned} (y_{15}, \alpha_{15}, \beta_{15})_{LR} = \\ = (a_{91}x_{11} + a_{92}x_{12} + b_{91}u_5 + b_{92}u_6 + a_{90}, \\ b_{91}\gamma_5 + b_{92}\gamma_6, b_{91}\phi_5 + b_{92}\phi_6)_{LR} \end{aligned} \quad (91)$$

$$\begin{aligned} (y_{16}, \alpha_{16}, \beta_{16})_{LR} = \\ = (a_{10.1}x_{11} + a_{10.2}x_{12} + b_{10.1}u_5 + b_{10.2}u_6 + \\ + a_{10.0}, b_{10.1}\gamma_5 + b_{10.2}\gamma_6, b_{10.1}\phi_5 + b_{10.2}\phi_6)_{LR} \end{aligned} \quad (92)$$

For the ethylene yield equation, the following $(L-R)$ -representation was obtained:

$$\begin{aligned} (y_{18}, \alpha_{18}, \beta_{18})_{LR} = \\ = (a_{11.1}x_{17} + a_{11.2}x_{17}^2 + a_{11.3}x_{17}x_{16} + b_{11.1}x_{17}u_7 + \\ + b_{11.2}x_{17}u_8 + b_{11.3}x_{16}u_8 + b_{11.4}u_7 + a_{11.0}, a_{11.3}x_{17}\alpha_{16} + \\ + b_{11.3}u_8\alpha_{16}, a_{11.3}x_{17}\beta_{16} + b_{11.3}u_8\beta_{16})_{LR} \end{aligned} \quad (93)$$

Introducing the following notations:

$$\begin{aligned} F &\triangleq y_{18} \\ \alpha_F &\triangleq \alpha_{18} \\ \beta_F &\triangleq \beta_{18} \\ \tilde{y}_{15} &\triangleq (y_{15}, \alpha_{15}, \beta_{15})_{LR} \\ \tilde{y}_{16} &\triangleq (y_{16}, \alpha_{16}, \beta_{16})_{LR} \\ \tilde{y}_{15} &\triangleq (y_{15}, \alpha_{15}, \beta_{15})_{LR} \\ \tilde{y}_{15} &\triangleq (\bar{y}_{15}, \bar{\alpha}_{15}, \bar{\beta}_{15})_{LR} \\ \tilde{y}_{16} &\triangleq (\bar{y}_{16}, \bar{\alpha}_{16}, \bar{\beta}_{16})_{LR} \\ \tilde{y}_{16} &\triangleq (y_{16}, \alpha_{16}, \beta_{16})_{LR} \\ \tilde{y}_{18} &\triangleq (y_{18}, \alpha_{18}, \beta_{18})_{LR} \\ \tilde{u}_5 &\triangleq (u_5, \gamma_5, \phi_5)_{LR} \\ \tilde{u}_6 &\triangleq (u_6, \gamma_6, \phi_6)_{LR} \end{aligned}$$

The constraints system (60)–(90) is the rewritten as follows:

$$(F, \alpha_F, \beta_F)_{LR} = (y_{18}, \alpha_{18}, \beta_{18})_{LR} \rightarrow \max \quad (94)$$

$$\begin{aligned} y_6 = a_{11}x_1 + a_{12}x_1^2 + a_{13}x_2 + a_{14}x_2^2 + a_{15}x_3 + \\ + a_{16}x_3^2 + a_{17}x_4 + a_{18}x_4^2 + a_{19}x_5 + a_{1,10}x_5^2 + \\ + b_{11}u_1 + b_{12}u_1^2 + b_{13}u_2 + b_{14}u_2^2 + a_{10} \end{aligned} \quad (95)$$

$$\begin{aligned} y_7 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 + \\ + b_{21}u_1 + b_{22}u_2 + a_{20} \end{aligned} \quad (96)$$

$$\begin{aligned} y_8 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 + \\ + b_{31}u_1 + b_{32}u_2 + a_{30} \end{aligned} \quad (97)$$

$$\begin{aligned} y_9 = a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 + \\ + b_{41}u_1 + b_{42}u_2 + a_{40} \end{aligned} \quad (98)$$

$$y_{10} = a_{51}x_8 + a_{52}x_9 + b_{51}u_3 + b_{52}u_4 + a_{50} \quad (99)$$

$$y_{11} = a_{61}x_8 + a_{62}x_9 + b_{61}u_3 + b_{62}u_4 + a_{60} \quad (100)$$

$$y_{12} = 100 - y_{11} \quad (101)$$

$$y_{13} = a_{71}x_8 + a_{72}x_9 + b_{71}u_3 + a_{70} \quad (102)$$

$$y_{14} = a_{81}x_8 + a_{82}x_9 + b_{81}u_3 + b_{82}u_4 + a_{80} \quad (103)$$

$$\begin{aligned} (y_{15}, \alpha_{15}, \beta_{15})_{LR} = (a_{91}x_{11} + a_{92}x_{12} + b_{91}u_5 + \\ + b_{92}u_6 + a_{90}, b_{91}\gamma_5 + b_{92}\gamma_6, b_{91}\phi_5 + b_{92}\phi_6)_{LR} \end{aligned} \quad (104)$$

$$\begin{aligned} (y_{16}, \alpha_{16}, \beta_{16})_{LR} = (a_{10.1}x_{11} + a_{10.2}x_{12} + b_{10.1}u_5 + \\ + b_{10.2}u_6 + a_{10.0}, b_{10.1}\gamma_5 + b_{10.2}\gamma_6, b_{10.1}\phi_5 + b_{10.2}\phi_6)_{LR} \end{aligned} \quad (105)$$

$$\begin{aligned} (y_{18}, \alpha_{18}, \beta_{18})_{LR} = (a_{11.1}x_{17} + a_{11.2}x_{17}^2 + a_{11.3}x_{17}x_{16} + \\ + b_{11.1}x_{17}u_7 + b_{11.2}x_{17}u_8 + b_{11.3}x_{16}u_8 + b_{11.4}u_7 + \\ + a_{11.0}, a_{11.3}x_{17}\alpha_{16} + b_{11.3}u_8\alpha_{16}, a_{11.3}x_{17}\beta_{16} + \\ + b_{11.3}u_8\beta_{16})_{LR} \end{aligned} \quad (106)$$

$$y_{19} = a_{12.1}x_{17} + b_{12.1}u_7 + b_{12.2}u_8 + a_{12.0} \quad (107)$$

$$\sum_{i=1}^4 x_i - y_7 - y_8 \geq S_1 \quad (108)$$

$$y_{10} + y_{13} - x_8 \leq S_2 \quad (109)$$

$$x_{10} - y_{17} \leq S_3 \quad (110)$$

$$x_{10} - y_{17} \geq 0 \quad (111)$$

$$y_{18} + y_{19} \leq x_{17} \quad (112)$$

$$\underline{y}_6 \leq y_6 \leq \bar{y}_6 \quad (113)$$

$$\underline{y}_{14} \leq y_{14} \leq \bar{y}_{14} \quad (114)$$

$$(y_{15}, \alpha_{15}, \beta_{15})_{LR} \lesssim (y_{15}, \alpha_{15}, \beta_{15})_{LR} \quad (115)$$

$$(y_{15}, \alpha_{15}, \beta_{15})_{LR} \lesssim (\bar{y}_{15}, \bar{\alpha}_{15}, \bar{\beta}_{15})_{LR} \quad (116)$$

$$(y_{16}, \alpha_{16}, \beta_{16})_{LR} \lesssim (y_{16}, \alpha_{16}, \beta_{16})_{LR} \quad (117)$$

$$(y_{16}, \alpha_{16}, \beta_{16})_{LR} \lesssim (\bar{y}_{16}, \bar{\alpha}_{16}, \bar{\beta}_{16})_{LR} \quad (118)$$

$$\underline{u}_j \leq u_j \leq \bar{u}_j, \forall j = \overline{1,4} \quad (119)$$

$$\underline{u}_7 \leq u_7 \leq \bar{u}_7 \quad (120)$$

$$\underline{u}_8 \leq u_8 \leq \bar{u}_8 \quad (121)$$

$$x_i = y_v, \forall i = v = \overline{7,10} \quad (122)$$

$$(x_{16}, \alpha_{16}, \beta_{16})_{LR} = (y_{16}, \alpha_{16}, \beta_{16})_{LR} \quad (123)$$

$$(x_{17}, \alpha_{17}, \beta_{17})_{LR} = (y_{17}, \alpha_{17}, \beta_{17})_{LR} \quad (124)$$

$$x_i \geq 0, \forall i = \overline{1,5} \quad (125)$$

$$y_v \geq 0, \forall v = \overline{6,14} \quad (126)$$

$$(y_{18}, \alpha_{18}, \beta_{18})_{LR} \gtrsim (0, \alpha_0, \beta_0)_{LR} \quad (127)$$

For these functions L and R , define the numbers:

$$\varepsilon_R = \sup\{u | R(u) = R(0) = 1\} \quad (128)$$

$$\delta_R = \inf\left\{u | u \geq 0, R(u) = \lim_{s \rightarrow \infty} R(s)\right\} \quad (129)$$

$$\varepsilon_L = \sup\{u | L(u) = L(0) = 1\} \quad (130)$$

$$\delta_L = \inf\left\{u | u \geq 0, L(u) = \lim_{s \rightarrow \infty} L(s)\right\} \quad (131)$$

The constraints can be written as follows:

$$\varepsilon_R(\beta_{15} - \beta_{15}) \leq y_{15} - \underline{y}_{15} \quad (132)$$

$$\delta_R(\beta_{15} - \beta_{15}) \leq y_{15} - \underline{y}_{15} \quad (133)$$

$$\varepsilon_L(\alpha_{15} - \alpha_{15}) \leq y_{15} - \underline{y}_{15} \quad (134)$$

$$\delta_L(\alpha_{15} - \alpha_{15}) \leq y_{15} - \underline{y}_{15} \quad (135)$$

$$\varepsilon_R(\beta_{15} - \bar{\beta}_{15}) \leq \bar{y}_{15} - y_{15} \quad (136)$$

$$\delta_R(\beta_{15} - \bar{\beta}_{15}) \leq \bar{y}_{15} - y_{15} \quad (137)$$

$$\varepsilon_L(\bar{\alpha}_{15} - \alpha_{15}) \leq \bar{y}_{15} - y_{15} \quad (138)$$

$$\delta_L(\bar{\alpha}_{15} - \alpha_{15}) \leq \bar{y}_{15} - y_{15} \quad (139)$$

$$\varepsilon_R(\beta_{16} - \beta_{16}) \leq y_{16} - \underline{y}_{16} \quad (140)$$

$$\delta_R(\beta_{16} - \beta_{16}) \leq y_{16} - \underline{y}_{16} \quad (141)$$

$$\varepsilon_L(\alpha_{16} - \alpha_{16}) \leq y_{16} - \underline{y}_{16} \quad (142)$$

$$\delta_L(\alpha_{16} - \underline{\alpha}_{16}) \leq y_{16} - \underline{y}_{16} \quad (143)$$

$$\varepsilon_R(\beta_{16} - \bar{\beta}_{16}) \leq \bar{y}_{16} - y_{16} \quad (144)$$

$$\delta_R(\beta_{16} - \bar{\beta}_{16}) \leq \bar{y}_{16} - y_{16} \quad (145)$$

$$\varepsilon_L(\bar{\alpha}_{16} - \alpha_{16}) \leq \bar{y}_{16} - y_{16} \quad (146)$$

$$\delta_L(\bar{\alpha}_{16} - \alpha_{16}) \leq \bar{y}_{16} - y_{16} \quad (147)$$

$$\varepsilon_R(\beta_0 - \beta_{18}) \leq y_{18} \quad (148)$$

$$\delta_R(\beta_0 - \beta_{18}) \leq y_{18} \quad (149)$$

$$\varepsilon_L(\alpha_{18} - \alpha_0) \leq y_{18} \quad (150)$$

$$\delta_L(\alpha_{18} - \alpha_0) \leq y_{18} \quad (151)$$

Then the constraints system (94)–(127), taking into account the principle of constructing an analog for the fuzzy objective function extremum, can be represented as follows:

$$F = y_{18} \rightarrow \max \quad (152)$$

$$\alpha_F = \alpha_{18} \rightarrow \min \quad (153)$$

$$\beta_F = \beta_{18} \rightarrow \max \quad (154)$$

$$y_6 = a_{11}x_1 + a_{12}x_1^2 + a_{13}x_2 + a_{14}x_2^2 + a_{15}x_3 + a_{16}x_3^2 + a_{17}x_4 + a_{18}x_4^2 + a_{19}x_5 + a_{1,10}x_5^2 + b_{11}u_1 + b_{12}u_1^2 + b_{13}u_2 + b_{14}u_2^2 + a_{10} \quad (155)$$

$$y_7 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 + b_{21}u_1 + b_{22}u_2 + a_{20} \quad (156)$$

$$y_8 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 + b_{31}u_1 + b_{32}u_2 + a_{30} \quad (157)$$

$$y_9 = a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 + b_{41}u_1 + b_{42}u_2 + a_{40} \quad (158)$$

$$y_{10} = a_{51}x_8 + a_{52}x_9 + b_{51}u_3 + b_{52}u_4 + a_{50} \quad (159)$$

$$y_{11} = a_{61}x_8 + a_{62}x_9 + b_{61}u_3 + b_{62}u_4 + a_{60} \quad (160)$$

$$y_{12} = 100 - y_{11} \quad (161)$$

$$y_{13} = a_{71}x_8 + a_{72}x_9 + b_{71}u_3 + a_{70} \quad (162)$$

$$y_{14} = a_{81}x_8 + a_{82}x_9 + b_{81}u_3 + b_{82}u_4 + a_{80} \quad (163)$$

$$y_{15} = a_{91}x_{11} + a_{92}x_{12} + b_{91}u_5 + b_{92}u_6 + a_{90} \quad (164)$$

$$y_{16} = a_{10,1}x_{11} + a_{10,2}x_{12} + b_{10,1}u_5 + b_{10,2}u_6 + a_{10,0} \quad (165)$$

$$y_{18} = a_{11,1}x_{17} + a_{11,2}x_{17}^2 + a_{11,3}x_{17}x_{16} + b_{11,1}x_{17}u_7 + b_{11,2}x_{17}u_8 + b_{11,3}x_{16}u_8 + b_{11,4}u_7 + a_{11,0} \quad (166)$$

$$y_{19} = a_{12,1}x_{17} + b_{12,1}u_7 + b_{12,2}u_8 + a_{12,0} \quad (167)$$

$$\alpha_{15} = b_{91}\gamma_5 + b_{92}\gamma_6 \quad (168)$$

$$\alpha_{16} = b_{10,1}\gamma_5 + b_{10,2}\gamma_6 \quad (169)$$

$$\alpha_{18} = a_{11,3}x_{17}\alpha_{16} + b_{11,3}u_8\alpha_{16} \quad (170)$$

$$\beta_{15} = b_{91}\phi_5 + b_{92}\phi \quad (171)$$

$$\beta_{16} = b_{10,1}\phi_5 + b_{10,2}\phi \quad (172)$$

$$\beta_{18} = a_{11,3}x_{17}\alpha_{16} + b_{11,3}u_8\alpha_{16}, a_{11,3}x_{17}\beta_{16} + a_{11,3}u_8\beta_{16} \quad (173)$$

$$\sum_{i=1}^4 x_i - y_7 - y_8 \geq S_1 \quad (174)$$

$$y_{10} + y_{13} - x_8 \leq S_2 \quad (175)$$

$$x_{10} - y_{17} \leq S_3 \quad (176)$$

$$x_{10} - y_{17} \geq 0 \quad (177)$$

$$y_{18} + y_{19} \leq x_{17} \quad (178)$$

$$\underline{y}_6 \leq y_6 \leq \bar{y}_6 \quad (179)$$

$$\underline{y}_{14} \leq y_{14} \leq \bar{y}_{14} \quad (180)$$

$$\underline{y}_{15} - y_{15} \leq 0 \quad (181)$$

$$y_{15} - \bar{y}_{15} \leq 0 \quad (182)$$

$$\underline{y}_{16} - y_{16} \leq 0 \quad (183)$$

$$y_{16} - \bar{y}_{16} \leq 0 \quad (184)$$

$$\underline{\beta}_{15} - \beta_{15} \leq 0 \quad (185)$$

$$\beta_{15} - \bar{\beta}_{15} \leq 0 \quad (186)$$

$$\underline{\alpha}_{15} - \alpha_{15} \leq 0 \quad (187)$$

$$\alpha_{15} - \bar{\alpha}_{15} \leq 0 \quad (188)$$

$$\underline{\beta}_{16} - \beta_{16} \leq 0 \quad (189)$$

$$\beta_{16} - \bar{\beta}_{16} \leq 0 \quad (190)$$

$$\underline{\alpha}_{16} - \alpha_{16} \leq 0 \quad (191)$$

$$\alpha_{16} - \bar{\alpha}_{16} \leq 0 \quad (192)$$

$$\underline{u}_j \leq u_j \leq \bar{u}_j, \forall j = \overline{1,4} \quad (193)$$

$$\underline{u}_7 \leq u_7 \leq \bar{u}_7 \quad (194)$$

$$\underline{u}_8 \leq u_8 \leq \bar{u}_8 \quad (195)$$

$$x_i = y_v, \forall i = v = \overline{7,10} \quad (196)$$

$$x_i \geq 0, \forall i = \overline{1,5} \quad (197)$$

$$y_v \geq 0, \forall v = \overline{6,14} \quad (198)$$

$$\alpha_{18} \leq \alpha_0 \quad (199)$$

$$\beta_{18} \geq \beta_0 \quad (200)$$

Thus, mathematical models (152)–(200) have been obtained, describing a multi-criteria nonlinear vector optimization problem for a complex for commercial ethylene production under conditions of incomplete information. The solution to this problem can be obtained using known methods of vector optimization. Let us consider the solution of a crisp analog of the complex optimization problem using vector optimization methods. As shown earlier, the optimal control problem for the studied complex is reduced to a nonlinear multi-criteria problem. A distinguishing feature of this class of optimization problem is that quality of the solution is evaluated based on multiple criteria $y_1, y_2, \dots, y_m$, forming a vector criterion $y = (y_1, y_2, \dots, y_m)$, which is used to determine optimality conditions. In this case, the optimal solution x^0 is defined by two conditions: 1) the solution must be feasible, that is, belong to the set of admissible solutions X ; 2) the solution must be optimal, that is, it must optimize the efficiency vector y while considering the priority of the criteria.

Decision-making problems are described by the following optimization models:

$$x^0 = f^{-1}[\text{opt}(y(x), \lambda)]_{x \in X}$$

Or, in the case of non-uniqueness of the optimal solution—when the optimal subset of solutions X^0 is chosen as optimal—the model is written as:

$$X^0 = \{x^0\} = \bigcup_{x \in X} f^{-1}[\text{opt}(y(x), \lambda)]$$

where $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_m)$ is a priority vector that characterizes the relative importance of local criteria in the optimization problem; f^{-1} is the inverse mapping $y \rightarrow x = f^{-1}(y)$.

Problem (152)–(200) is a one-vector, finite-dimensional, single-scale convex problem with prioritized local criteria.

Table 3. Results of the optimal control problem for the ethylene production complex**Tabela 3.** Wyniki problemu optymalnego sterowania dla kompleksu produkcji etylenu

Parameters	Problem solution			
	with a crisp objective function	by the hard priority method	flexible priority method weight = 0.2	flexible priority method weight = 0.5
Temperature of the top K-10 [°C]	−94	−95	−93	−93
Temperature of the bottom K-10 [°C]	10	8	7	8
Propylene content in distillate K-11 [%]	0.441	0.289	0.395	0.280
Temperature of the top K-11 [°C]	−12	−14	−13	−14
Temperature of the bottom K-11 [°C]	71	71	74	72
Content of $\sum C_2$ hydrocarbons in the bottom product K-10 [%]	72,563	73.855	74.863	73.821
Temperature at the outlet of the R-2 reactor [°C]	104	104	105	100
Ethylene content in distillate K-11 [%]	83.868	84.455	81.313	77.226
Hydrogen- to-acetylene ratio in the EEF at the inlet of the R-2 reactor	3.408	3.409	3.658	2
Ethylene content in distillate K-10 [%]	0.614	0.233	0.101	0.295
Ethylene content at the outlet of P-2 [%]	77.435	79.548	77.935	80.3
Temperature of the top K-12,13 [°C]	−30	−28	−30	−30
Temperature of the bottom K-12,13 [°C]	−6	−6	−6	−6
The output of commercial ethylene from the ethylene fractionation column K-12,13 [t/h]	34.1	34.5	35.1	35.2

The results of solving this problem are presented in Table 3. The problem was solved using both hard and flexible priority methods, with two different weights.

Furthermore, the solution of this fuzzy problem was compared with the solution of a fuzzy problem that uses a crisp objective function. The hard priority principle gives good results when searching not for a single exact optimum, but for a certain range of solutions close to the optimum—a quasi-optimal set $\bar{X}^{0j}$.

An advantage of the hard priority method is that it does not require the specification of quantitative characteristics for the criteria priority λ or α , but only the order of the criteria in a priority series ($\alpha = (\alpha_1, \alpha_2, \dots, \alpha_m)$) is a weight vector, each component of which has the meaning of a weight coefficient determining the relative superiority of the j -th criterion over the rest).

The application of the flexible priority method requires specification of quantitative characteristics of priority λ or α , allowing only some preference to be given to the more important criteria when selecting a solution. In practice, this results in evaluating the solution quality using a weighted vector criterion – a vector pair (y, λ) – and modifying the optimality principle to transition from $opt y$ to $opt(y, \alpha)$. This correction of the optimality principle may be implemented in various ways. In the case of problem (152)–(200), the

correction is based on the principle of fair concession with priority, in this case:

$$opt(y, \alpha) = \max_{y \in Y} \sum_{j \in I} \alpha_j \log y_j$$

An important advantage of the flexible priority method is that it enables, within reasonable limits, the assignment of preference to more important criteria, taking into account their relative importance.

As demonstrated in the table of results pertaining to the solution of the optimal control problem of the technological complex for ethylene production using the methods of vector optimization, it is possible to achieve a decrease of the propylene content (concentration) in the distillate of the deethanization rectification column, which makes it possible to direct the amount of $\sum C_3$ hydrocarbons to the technological block for the production of commercial propylene and propane. In addition, a slight reduction in the temperatures of the rectification columns is achieved, leading to improved product purity and reduced thermal energy consumption for the process as a whole. Furthermore, based on the reduction in the ethylene content in the distillate of the demethanization column, it becomes possible to redirect the $\sum C_2$ hydrocarbons for further processing, resulting in an increase in the purified ethylene content in the ethane-ethylene fraction at the outlet of the acetylene hydrogenation reactor.

Results

A physically justified statement of the optimization problem for the technological complex for commercial ethylene production has been developed. It has been demonstrated that the optimal control problem for the studied complex is reduced to a mathematical programming problem with fuzzy parameters. A deterministic analog of a multidimensional nonlinear optimization problem with a fuzzy objective function and fuzzy constraints has been constructed. As a result, a multi-criteria vector optimization problem for control of a commercial ethylene production complex has been obtained. The applicability of various multi-criteria optimization methods to the solution of the obtained crisp analog of the fuzzy control optimization problem for the technological complex for commercial ethylene production has been studied. As shown by the obtained results, solving the fuzzy problem using any of the multi-criteria optimization methods yields an increase in ethylene output compared to the solution of a fuzzy problem with a crisp objective function, while also ensuring the physical validity of the solution.

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