

Allocation of capital investments of an oil company based on independent opportunity information

Alokacja inwestycji kapitałowych w spółce naftowej na podstawie niezależnych informacji o możliwościach inwestycyjnych

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ABSTRACT: To address the problem of investment allocation while optimizing the total return on investment for projects under risk conditions associated with uncertainty of project income, a linear combination with non-negative coefficients of two loss-making models was employed: the standard minimax loss-making model and the minimax loss-making level model. The resulting probabilistic programming problem, under the assumption that the probabilistic variables characterizing project profitability are independent (unrelated), is reduced to a linear programming problem. Solving this problem using the simplex method yields a vector of the equity distribution of investments that determines the total return on investment according to the criterion of minimizing the loss function. Based on the proposed approach to the optimal solution of the investment allocation problem, an algorithm for project investment planning was developed using a database of invested and returned funds from previous planning periods, according to which distributions of possible investment returns are modeled. The forecast of total investments in the new planning period is presented as the sum of the basic (main) portion and the additional portion of the investment. The first portion is determined using the projected price of each project's products in the new planning period and the planned (in case of over-fulfillment) or actual (in case of under-fulfillment) volume of products in the previous planning period. The second portion is determined using the equity investment vector obtained from the solution of the linear programming problem, applied to the total cost of project revenues in the previous planning period, evaluated at projected prices in the new planning period. For a project with no income, the cost of income is considered zero. The numerical implementation of the algorithm is illustrated using the example of investment planning in an oil company for oil and gas production projects.

Keywords: probabilistic programming, investments, investment portfolio, minimax criterion, loss-making function, return on investment, income, concentrated distribution, probability distribution, measure of opportunity, measure of necessity, fractal model, random variable, fuzzy variable membership function.

STRESZCZENIE: W celu rozwiązania problemu alokacji inwestycji przy jednoczesnej optymalizacji całkowitego zwrotu z inwestycji dla projektów realizowanych w warunkach ryzyka związanego z niepewnością dochodu z projektu, zastosowano liniową kombinację z nieujemnymi współczynnikami dwóch modeli generowania strat: standardowego modelu minimax (minimalizowania maksymalnych możliwych strat) oraz modelu minimalizacji maksymalnego poziomu strat. Powstały w ten sposób problem programowania probabilistycznego, przy założeniu, że zmienne probabilistyczne charakteryzujące rentowność projektu są niezależne (niepowiązane), sprowadza się do problemu programowania liniowego. Rozwiązanie tego problemu metodą simpleksową daje wektor rozkładu kapitału inwestycyjnego, który określa całkowity zwrot z inwestycji zgodnie z kryterium minimalizacji funkcji straty. W oparciu o zaproponowane podejście do optymalnego rozwiązania problemu alokacji inwestycji, opracowano algorytm planowania inwestycji projektowych z wykorzystaniem bazy danych dotyczących nakładów i zwrotów inwestycyjnych z poprzednich okresów planistycznych, według której modelowane są rozkłady możliwych zwrotów z inwestycji. Prognoza całkowitych inwestycji w nowym okresie planistycznym przedstawiona jest jako suma podstawowej (głównej) części inwestycji oraz części dodatkowej. Pierwsza część określana jest na podstawie prognozowanej ceny produktów każdego projektu w nowym okresie planistycznym oraz planowanego (w przypadku nadwykonania) lub rzeczywistego (w przypadku niewykonania) wolumenu produkcji w poprzednim okresie planistycznym. Druga część wyznaczana jest przy użyciu wektora alokacji kapitału uzyskanego z rozwiązania problemu programowania liniowego, zastosowanego do całkowitej wartości przychodów projektów w poprzednim okresie planistycznym, przeliczonej według prognozowanych cen w nowym okresie. Dla projektów, które nie generują dochodu, koszt dochodu przyjmuje się jako zerowy. Numeryczna implementacja algorytmu została zilustrowana na przykładzie planowania inwestycji w spółce naftowej dla projektów wydobycia ropy i gazu.

Słowa kluczowe: programowanie probabilistyczne, inwestycje, portfel inwestycyjny, kryterium minimaksowe, funkcja strat, zwrot z inwestycji, dochód, rozkład skoncentrowany, rozkład prawdopodobieństwa, miara możliwości, miara konieczności, model fraktalny, zmienna losowa, funkcja przynależności zmiennej rozmytej.

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Introduction

When planning investments, the central task is to assess their profitability and risk level. Economic risk is understood as “the danger, possibility of damage” (Dyshin et al., 2023), i.e. loss of part of enterprise’s resources, loss of income, or the occurrence of additional expenses as a result of production or financial activities.

The risk of accepting investment projects is a consequence of unavoidable information uncertainty. The possibility of risk, i.e., a situation in which a project considered viable during development turns out to be unprofitable during implementation, arises when the parameters achieved during the investment process deviate from the planned values, or when certain factors were not taken into account. Nevertheless, the investor is obliged to attempt to measure the risks of investment decisions both at the stage of project development and during implementation. The use of any type of probabilities in investment analysis becomes less justified when the parameters in question are statistically underdetermined or when expert activity and the quality of their forecasts are low. Therefore, the application of fuzzy mathematics, based on fuzzy information regarding changes in external conditions, appears to be more aligned with market realities and more convenient from a computational perspective (Habibov et al., 2023).

In the well-known Markovitz model of investment distribution (Nedosekin, 2000), the investor’s expected return is treated as a random variable. Based on this model, the problem of investment portfolio allocation (portfolio selection) is solved using stochastic programming methods, according to the criterion minimizing the variance of the total expected return, under the assumption that the maximum average return is preferable. However, this model allocates investments to projects whose expected returns are entirely dependent on one another. Moreover, the traditional theory of investment portfolio allocation incorporates risk minimization, which is addressed through probabilistic programming methods.

Probabilistic programming is analogous to stochastic programming. Consequently, its application to the investment allocation problem yields expected results when project returns are treated as possible variables. Probabilistic programming offers two main advantages (Altunin and Semukhin, 2005):

1. it readily incorporates expert information in income estimation;
2. it simplifies the treatment of risk minimization compared to stochastic programming. However, in most existing approaches using probabilistic programming, independent random variables are assumed, which has hindered its application to investment allocation.

Altunin and Semukhin (2005) proposed a method of proba-

bilistic programming using independent random variables is proposed, making it suitable for solving the investment allocation problem. A brief overview of fuzzy programming methods for solving investment portfolio allocation problems is provided, along with a comparison to the corresponding stochastic programming methods.

A method of probabilistic programming for solving the investment portfolio allocation problem (Markovitz, 1959) is based on the minimax loss rate model. This model results in the distribution of investments across projects under conditions of uncertainty regarding their profitability in the planning period, according to the minimization of the loss function $r(\mathbf{x}, \mathbf{c})$. This function characterizes the measure of unprofitability of the total investment $\mathbf{c}^T \mathbf{x}$ (T denotes transposition), where the decision in the planned period is represented by the investment distribution vector $\mathbf{c} = (c_1, \dots, c_n)^T$ for projects 1, 2, ..., n , and the share vector $\mathbf{x} = (x_1, \dots, x_n)^T$, $\mathbf{x} \geq \mathbf{0}$, $\sum_{i=1}^n x_i = \mathbf{1}$. Any concentrated solution can be unprofitable. Since the model minimizes the worst-case loss (Markovitz, 1959), the resulting solution is considered the most preferable. Some variants of this model were proposed earlier (Inuiguchi and Ramik 2000, Inuiguchi and Tanino, 2000). In the first variant (Inuiguchi and Ramik, 2000), the usual minimax loss (minimax regret) model is considered, in which the greatest loss from the distribution of the investment vector $\mathbf{c} = (c_1, \dots, c_n)$ according to the equity vector $\mathbf{x} = (x_1, \dots, x_n)$ is defined as the difference between the optimal total return with the distribution vector \mathbf{c} and the resulting total return level is $\mathbf{c}^T \mathbf{x}$. In the second variant, the loss rate model (regret rate model) is used, which is equivalent to the success rate model proposed by Inuiguchi and Tanino (2000). In the unprofitability level model, the maximum loss at return level $\mathbf{c}^T \mathbf{x}$ is defined as the ratio of the difference between the optimal total return level with the investment vector \mathbf{c} and the resulting total return level $\mathbf{c}^T \mathbf{x}$ to the optimal total return level (income rate).

Markovitz (1959) provided a description of the general case of a loss function satisfying certain properties. It is shown that, in the two variants mentioned above, the redundancy function $r(\mathbf{x}, \mathbf{c})$ cannot determine the optimal total return on investment due to the negativity of the function $f(r_1)$, included in the definition of the function $r(\mathbf{x}, \mathbf{c})$, for all $r_1 \in [L, R]$. L and R are defined as $L = \min_{i=1,2,\dots,n} c_i^L(0)$, $R = \min_{j(I=1,2,\dots,n)} R = \min_{i=1,2,\dots,n} c_i^R(0)$; $c_i^L(0)$ and $c_i^R(0)$ are values at the level $h = 0$ of the values $c_i^L(h) = \inf\{q | \pi_{c_i}(q) > h\}$, $c_i^R(h) = \sup\{q | \pi_{c_i}(q) > h\}$, π_{c_i} is the probability distribution of the fuzzy quantity c_i . To eliminate this drawback Markovitz (1959), recommended to use a non-negative linear combination of the options (Inuiguchi and Tanino, 2000), although the method for the optimal construction of such a linear combination is not provided.

It should be noted that previously existing probabilistic programming methods applied to the problem of investment portfolio allocation produced only a concentrated solution, in which one and only one project i_0 was selected, i.e. $x_i = 1$ and $i = i_0$ for $i \neq i_0$; $i, i_0 \in \{1, \dots, n\}$.

Objective

To construct a non-negative linear combination of the two minimax loss models described above, resulting in the distribution of the investment vector $c = (c_1, \dots, c_n)$ with the equity vector $x = (x_1, \dots, x_n)$, such that the total investment $c^T x$ yields the minimum value of the loss function $r(x, c)$. It is assumed that $e^T x = 1, x \geq 0, e = (e_1, \dots, e_n)^T$, and that the investment random variables $c_i (i = 1, \dots, n)$ are independent and normally distributed with mean m_i and variance σ_i^2 . The numerical implementation of the resulting linear combination is demonstrated by the example of the annual capital investment distribution of an oil company for oil and gas projects.

Statement of the problem of investment planning

Consider the problem of planning capital investments by a company in projects $i = 1, 2, \dots, n$ with known information on investments c_{ij}^{pl} in previous planning periods $j = 1, \dots, m$, and their returns c_{ij}^f , with returns defined as $\Delta c_{ij} = c_{ij}^f - c_{ij}^{pl}$. Based on statistical information about the yields of Δc_{ij} from previous planning periods and expert information about Δc_{ij} in the new $(m+1)$ planning period, it is possible to construct the random variables for additional (compared to the last m -planning period) investments c_1, \dots, c_n in $(m+1)$ -th planning period with a share vector $x = (x_1, \dots, x_n), \sum_{i=1}^n x_i = 1, x \geq 0$. Then, the problem of deciding on the maximum return on investments can be formulated as follows:

$$\max cx$$

subject to the constraint:

$$e^T x = 0, x \geq 0, e = (e_1, \dots, e_n) \tag{1}$$

where $C = (C_1, \dots, C_n)^T$ is a vector of independent random variables satisfying a multidimensional probability distribution:

$$\pi_C(c) = \min_{i=1,2,\dots,n} \pi_{C_i}(c_i) \tag{2}$$

where π is a probability measure.

Generally, c_i may correlate with $c_j (i \neq j)$, but we assume that c_i is independent of $c_j (i \neq j)$ for all $(i, j), i \neq j$, in order to highlight the essential differences between stochastic and fuzzy programming. In addition, we assume that the investment rates c_i follow a normal distribution law $N(m_i, \sigma_i^2)$, where m_i and σ_i^2 are the mean and variance of the variable c_i .

Given a priori level $h^0 \in (0, 1]$, problem (1) can be formulated as a problem of maximizing an auxiliary (fractal) variable z , such that the necessity measure of the event where the value

of the objective function in (1) is no less than z is at least h^0 . This is the so-called fractile model (Markowitz, 1959; Altunin and Semukhin, 2005):

$$\max z$$

subject to the constraints:

$$N_c(\{c | c^T x \geq z\}) \geq h^0 \tag{3}$$

$$e^T x = 1, x \geq 0$$

where: N_c is the necessity measure for a possible distribution of π_c , defined as (Markovitz, 1959):

$$(D) = \begin{cases} \inf_{c \in D} (1 - \pi_c(c)) & \text{if } D \text{ is not a universal set} \\ 1 & \text{if } D \text{ is a universal set} \end{cases} \tag{4}$$

The mayor $N_c(\{c | c^T x \geq z\})$ shows the degree of necessity of fulfilling the condition that the objective function in (1) is not less than z and is represented as:

$$N_C(\{c | c^T x \geq z\}) = \inf_{c | c^T x < z} (1 - \pi_C(c)) \tag{5}$$

Given (2) and the assumption of independence of the random variables c_i , the problem (3) is reduced to the following linear programming problem (Inuiguchi and Ramik, 2000):

$$\max (c^L (1 - h^0))^T x$$

subject to:

$$e^T x = 1, x \geq 0 \tag{6}$$

where:

$$c^L(\cdot) = (c_1^L(\cdot), c_2^L(\cdot), \dots, c_n^L(\cdot))^T \tag{7}$$

with the functions $c_n^L(h)$, defined in the introduction.

Solving the problem based on the minimax loss rate model

We assume that the investor receives information about the rate of return c_i of the i -th investment only after distributing investments across projects $i (i = 1, 2, \dots, n)$ in accordance with an acceptable solution to problem $x(1)$. Then, the degree of loss, i.e., the function $r(x, c)$, can be quantified by the expression:

$$r(x, c) = \max_{y | e^T y = 1, y \geq 0} F(c^T y, c^T x) \tag{8}$$

where F is a function from $D_1 \times D_2$ to $R (D_1, D_2 \leq R, R)$ is the one-dimensional Euclidean space, i.e. the real axis, and is a continuous function such that $F(\cdot; r)$ is strictly increasing and $F(r; \cdot)$ is strictly decreasing. The continuity of the function F implies the continuity of the function $r(x, c)$ in both variables.

At the moment of making a decision, the investor cannot know in advance the return rate c but does know the probability distribution $\pi_C(c)$. According to the generalization principle (Inuiguchi and Tanino, 2000), the probability distribution of the loss value $\pi_{R(x)}$ can be defined according to

(Markowitz (1959):

$$\pi_{R(x)}(r) = \sup_{c|r=r(x;c)} \pi_C(c) \quad (9)$$

which makes it possible to consider the investment distribution probability problem (1) as a problem of minimizing the value of $R(x)$ given the probability distribution $\pi_{R(x)}$

$$\min R(x)$$

subject to:

$$e^T x = 1, x \geq 0 \quad (10)$$

Using the fractal model (3), problem (10) is written as:

$$\min z$$

subject to:

$$\begin{aligned} N_{R(x)}(\{r|r \geq z\}) &\geq h^0 \\ e^T x = 1, x &\geq 0 \end{aligned} \quad (11)$$

Problem (11) is equivalent to the following minimax problem with linear constraints (5):

$$\min z$$

subject to:

$$\begin{aligned} \max_{\substack{c,y \\ c \in cl(C)_{1-h^0} \\ e^T y = 1, y \geq 0 \\ e^T x = 1, x \geq 0}} F(c^T y, c^T x) &\leq z \end{aligned} \quad (12)$$

where F is the function from the right side of expression (8), and the closure $cl(C)_{1-h^0}$ is defined by the formula:

$$\begin{aligned} cl(C)_{1-h^0} &= \\ = \left\{ c = (c_1, c_1, \dots, c_n) \mid c_i^L(1-h^0) \leq c_i \leq c_i^R(1-h^0), i = 1, 2, \dots, n \right\} \end{aligned} \quad (13)$$

where the functions $c_i^L(h)$ and $c_i^R(h)$ are defined in the introduction.

According to theorem (1) (Markowitz, 1959), the optimal solution to problem (1) is a concentrated investment distribution with the maximum total return, namely the solution $x_C = 1$, $x_i = 0$, $i \neq i_0$, where $c_{i_0} \geq c_i$, $i = 1, 2, \dots, n$. Taking this theorem and the properties of the function F from expression (8) into account, the following relation holds:

$$\begin{aligned} \max_{\substack{c,y \\ c \in cl(C)_{1-h^0} \\ e^T y = 1, y \geq 0}} F(c^T y, c^T x) &= \\ = \max_{c \in cl(C)_{1-h^0}} F \left(\max_{e^T y = 1, y \geq 0} c^T y, c^T x \right) &= \\ = \max_{i=1,2,n,n} \max_{c \in cl(C)_{1-h^0}} F(c_i, c^T x) \end{aligned} \quad (14)$$

Therefore, problem (12) is reduced to:

$$\min z$$

subject to:

$$\begin{aligned} \max_{c \in cl(C)_{1-h^0}} F(c_i, c^T x) &\leq z, i = 1, 2, \dots, n \\ e^T x = 1, x &\geq 0 \end{aligned} \quad (15)$$

Now consider the case when the function F is given by:

$$F(r_1, r_2) = \varphi(f(r_1)r_2 + g(r_1)) \quad (16)$$

where φ is a strictly increasing function from R to R , and f and g are functions from D_1 to R and from D_2 to R , respectively, satisfying the following properties:

- a) $f(r) < 0, \forall r \in [L, R]$
 $f'(r_1)r_2 + g'(r_1) > 0, \forall (r_1, r_2) \in [L, R] \times [L \times R]$ (17)
- b) for everyone
 $i \in \{1, 2, \dots, n\} \inf_{\substack{c,x \\ c \in (C)_0 \\ e^T x = 1, x \geq 0}} (f'(c_i)c^T x + g'(c_i) + f(c_i)x_i) \geq 0$

where $(C)_0 = (C)_{1-h^0|_{(h^0=1)}}$, and the values L and R are defined in the introduction.

When F is defined as in (16), problem (15) can be written as (Markowitz, 1959):

$$\min q$$

subject to:

$$\begin{aligned} f \left(c_i^R(1-h^0) \right) \left(\sum_{\substack{j=1 \\ j \neq i}}^n c_j^L(1-h^0)x_j + c_i^R(1-h^0)x_i \right) &\leq \\ &\leq q - g \left(c_i^R(1-h^0) \right) \\ i = 1, 2, \dots, n \end{aligned} \quad (18)$$

$$e^T x = 1, x \geq 0$$

Problem (18) is a linear programming problem involving $n+1$ non-negative variables (q, x_1, \dots, x_n) with $n+1$ constraints of the type \leq . Even when solved using the simplex method (18), the result will be the equity vector $x = (x_1, \dots, x_n)$ representing the investment distribution.

The functions $f(r)$, $g(r)$ and $\varphi(r)$, included in the representation (16) for the variant from (Inuiguchi and Ramik, 2000) of the minimax loss model take the following forms (Markowitz, 1959):

$$f(r) = -1 \quad g(r) = r \quad \varphi(r) = r \quad (19)$$

while for the variant from (Inuiguchi and Tanino, 2000):

$$f(r) = -\frac{1}{1+r} \quad g(r) = \frac{r}{1+r} \quad \varphi(r) = r \quad (20)$$

For a linear combination of these two variants with coefficients $\lambda_1 \geq 0, \lambda_2 \geq 0$ Markowitz (1959) proposed the following solution:

$$f(r) = -\frac{\lambda_1(1+r) + \lambda_2}{1+r}, \quad g(r) = \frac{\lambda_1 r^2 + (\lambda_1 + \lambda_2)r}{1+r}, \quad \varphi(r) = r \quad (21)$$

however, the method of selecting the coefficients λ_1, λ_2 is not specified in (19).

Below is the approach to the optimal choice of coefficients λ_1, λ_2 for the non-negative linear combination of the variants proposed by Inuiguchi and Ramik (2000) and Inuiguchi and Tanino (2000).

Let the function $F(r_1, r_2)$ be given by (16), where f, g and φ are expressed by formulas (21). Then, using the notation:

$$\begin{aligned} r_i^R &= C_i^R(1-h^0), \quad r_i^L = C_i^L(1-h^0), \\ f(r_i^R) &= -\frac{\lambda_1(1+r_i^R) + \lambda_2}{1+r_i^R}, \\ g(r_i^R) &= \frac{\lambda_1(r_i^R)^2 + (\lambda_1 + \lambda_2)r_i^R}{1+r_i^R} \end{aligned} \quad (22)$$

Optimization problem (18), with the introduction of auxiliary variables ξ_1, \dots, ξ_{n0} can be written as:

$$\min q$$

subject to:

$$\begin{aligned} f(r_i^R) \left(\sum_{j=1}^n r_i^L x_j + r_i^R x_i \right) + g(r_i^R) \sum_{j=1}^n x_j + i - q, \quad (i=1, \dots, n) \\ \lambda_1 x_i \geq 0, \quad \lambda_2 x_i \geq 0, \quad \xi_i \geq 0, \quad (i=1, \dots, n) \end{aligned} \quad (23)$$

Using the notation $N = 3n + 1$

$$X_i = \lambda_1 x_i \quad X_{n+i} = \lambda_2 x_i \quad X_{2n+i} = \xi_i \quad X_N = q \quad (i=1, \dots, n) \quad (24)$$

Problem (23), in terms of the non-negative variables X_1, X_2, \dots, X_N , becomes a linear programming problem:

$$\begin{aligned} \min X_N \\ -r_i^L \sum_{j=1}^n \left(X_j + \frac{1}{1+r_i^R} X_{(n_0+j)} \right) - r_i^R \left(X_i + \frac{1}{1+r_i^R} X_{(n_0+i)} \right) + \\ + r_i^R \sum_{j=1}^n \left(X_j + \frac{1}{1+r_i^R} X_{(n_0+j)} \right) + X_{(2n_0+i)} - X_n = 0, \quad (i=1, n, n) \end{aligned} \quad (25)$$

Applying the simplex method to problem (25) yields the solution $X_j (j=1, \dots, N)$. Since $\lambda_1 = \sum_{i=1}^n X_i$ and $\lambda_2 = \sum_{i=1}^n X_{n+i}$, it follows that $x_i = X_i/\lambda_1$ or, equivalently $x_i = X_{n+i}/\lambda_2 (i=1, \dots, n)$.

Thus $x = (x_1, \dots, x_n)$ is the share vector of additional investments $c = (c_1, \dots, c_n)$.

Let Q_{ij}^{pl}, Q_{ij}^f be the planned and actual production volumes of the i -th project in the j -th planning period and S_{ij}^{pl}, S_{ij}^f the corresponding monetary funds expressed in terms of prices using coefficients P_{ij} :

$$S_{ij}^{pl} = Q_{ij}^{pl} \cdot P_{ij}, \quad S_{ij}^f = Q_{ij}^f \cdot P_{ij} \quad (26)$$

and through $C_{ij} = S_{ij}^f - S_{ij}^{pl}$, we denote the corresponding investor's income (with $S_{ij}^f < S_{ij}^{pl}$, C_{ij} characterizes the investor's loss), which are the values of the random variables C_i from (2).

We introduce the values:

$$\tilde{Q}_{i,m} = \begin{cases} Q_{i,m}^f & \text{if } Q_{i,m}^f \leq Q_{i,m}^{pl} \\ Q_{i,m}^{pl} & \text{if } Q_{i,m}^f > Q_{i,m}^{pl} \end{cases} \quad (27)$$

$$\tilde{S}_{i,m+1} = \tilde{Q}_{i,m} \cdot P_{i,m+1} \quad (28)$$

where: $P_{i,m+1}$ is the projected product price of the i -th project for the $(m+1)$ -th planning period.

Assuming that the investor can invest in production in the $(m+1)$ -th period the total income earned in the previous m -th period from all projects under consideration, calculated using the projected prices of $P_{i,m+1}$, i.e. using the funds:

$$S_{m+1} = \sum_{i=1}^n (Q_{i,m}^f - \tilde{Q}_{i,m}) \cdot P_{i,m+1} \quad (29)$$

as additional investments in the $(m+1)$ -th period. In other words, it is assumed that the total income from the m -th period can be reinvested according to the equity vector $x = (x_1, \dots, x_n)$ obtained from the solution of problem (25), allocating additional investments $x = (x_1, \dots, x_n)$ to each i -project. Then, the forecast values of total investment in each i -th project in the $(m+1)$ -th period, $S_{i,m+1}^{tot}$, can be expressed as:

$$S_{i,m+1}^{tot} = \tilde{S}_{i,m+1} + S_{m+i} \cdot x_i \quad (30)$$

We assume that C_i are fuzzy normal variables in the sense of Nahmias (1978) and Inuiguchi et al. (1993) with probability distributions centered at c_i^c and span w_i :

$$\pi_{C_i}(q) = \exp\left(-\frac{(q - c_i^c)^2}{w_i}\right) \quad (31)$$

To estimate the parameters c_i^c of the distribution (31), we use the results of c_i^c (Dubois and Prade, 1980) obtained for normal fuzzy variables in the sense of Nahmias (1978) and Hong (2001).

Let X be a normal fuzzy variable defined on the image space $(\Gamma, \mathcal{Y}, \sigma)$, where Γ is the base set; \mathcal{Y} is the class of all subsets of the set Γ , and σ is a scale measure satisfying the properties: (i) $\sigma(\emptyset) = 1$ and $\sigma(\Gamma) = 1$; (ii); (ii) for an arbitrary union (finite, countable or uncountable) of sets A_α by $\mathcal{Y} \sigma(\cup_\alpha A_\alpha) = \sup_\alpha \sigma(A_\alpha)$.

The membership function of the fuzzy variable X (denoted μ_X) maps from R (real numeric line) to the unit interval $[0, 1]$, given by the formula (Nahmias, 1978):

$$\mu_X(x) = \sigma(\gamma : X(\gamma) = x) \text{ for all } x \in R$$

The value of $\mu_X(x)$ at point x can be interpreted as the possibility of the equality $X = x$ being fulfilled (Cai, 1993), although this does not align with the definition of the probability measure by Zadeh (1978). Thus, the probability distribution

function of a fuzzy variable X (denoted π_x or μ_x) maps from R to $[0, 1]$, defined by the formula:

$$\pi_x(x) = \mu_x(x) = \sigma(X = x) \text{ for all } x \in R$$

For a normal fuzzy variable defined on the image space $(\Gamma, \mathcal{Y}, \sigma)$ we have:

$$\mu_X(x) = \sigma(X = x) = \exp\left[-\left(\frac{x-a}{b}\right)^2\right]$$

Let $X_{j,i}$ be the j -th observation of variable X at time t_j , and X_1, \dots, X_N be independent variables with normal distribution of $N(a, b)$ and x_1, \dots, x_N of their implementation.

Given the possibility α , we define ε_α such that:

$$\begin{aligned} \sigma\left(\left|\frac{X_j - a}{b}\right| > \varepsilon_\alpha, i = 1, \dots, n\right) &= \\ = \min_{1 \leq j \leq N} \sigma\left(\left|\frac{X_j - a}{b}\right| > \varepsilon_\alpha\right) &= \alpha \end{aligned} \quad (32)$$

Then, for risk level α , the following estimates of parameters a and b hold (Nahmias, 1978):

$$\hat{a} = \frac{1}{2} \left(\max_{1 \leq j \leq N} x_j + \min_{1 \leq j \leq N} x_j \right) \quad (33)$$

$$\hat{b} = \left(\max_{1 \leq j \leq N} x_j - \min_{1 \leq j \leq N} x_j \right) / 2\varepsilon_\alpha \quad (34)$$

In calculations, we take $\alpha = c^{-4}$ and therefore, $\varepsilon_\alpha = 2$.

Study results

Consider the distribution of investments of the SOCAR oil company (State Oil Company of the Azerbaijan Republic) across the following projects: $i = 1$ – AZNEFT (oil); $i = 2$ – JV/LTDD (oil); $i = 3$ – AZNEFT (gas); $i = 4$ – JV/LTDD (gas).

Let Q_{ij}^{pl} and Q_{ij}^a be the planned and actual production volumes for the i -th project in the j -th year ($j = 1, 2, \dots, m$; $m = 2$; $j = 1$ – 2017, $j = 2$ – 2018, $j = 3$ – 2019, $j = 4$ – 2020). Oil volumes are expressed in thousand tons, and gas volumes in cubic meters (m^3) (see Table 1). In Table 2, oil and gas prices

Table 2. Oil and gas prices

Years	Oil [\$/barrel]	Gas [\$/1000 m ³]
2017	52.1	180.0
2018	73.3	185.2
2019	60.8	285.5
2020	43.7	443.3
2021	71.6	846.3

are set: for oil – in \$/barrel, for gas – in \$/1000 m³ (1 ton of oil \approx 6.6 barrels).

Let $d_j^{(oil)}$ be the oil price in \$/barrel in year j , and $d_j^{(gas)}$ be the price of 1000 m³ of gas in year j in \$. The price of 1000 tons of oil in the j -th year, $P_j^{(oil)}$, will be written as $P_j^{(oil)} = 6.6 \cdot 10^3 d_j^{(oil)}$, and for 1000 m³ gas $P_j^{(gas)}$ – in the form of $P_j^{(gas)} = d_j^{(F)}$. Then, the invested funds in the j -th year for projects $i = 1, 2$ will be written as $S_{ij}^{pl} = Q_{ij}^{pl} \cdot P_j^{(oil)}$, and for projects $i = 3, 4$ – $S_{ij}^{pl} = Q_{ij}^{pl} \cdot P_j^{(gas)}$.

Let $c_{ij} = S_{ij}^f - S_{ij}^{pl}$ be the investor's income for project i in year j (with $S_{ij}^f < S_{ij}^{pl}$ representing the investor's loss). Based on the data in Tables 1 and 2, we obtain:

$$\begin{aligned} c_{11} &= 3606513.6 \cdot 10^3, c_{12} = 5145014.8 \cdot 10^3, c_{13} = 4304394.0 \cdot 10^3 \\ c_{14} &= 2995769 \cdot 10^3, c_{21} = 707846.11 \cdot 10^3, c_{22} = 1057968.8 \cdot 10^3 \\ c_{23} &= 884986.9 \cdot 10^3, c_{24} = 636084.34 \cdot 10^3, c_{31} = 1807052.4 \cdot 10^3 \\ c_{32} &= 1782624.0 \cdot 10^3, c_{33} = 2869728.8 \cdot 10^3, c_{34} = 4463029.0 \cdot 10^3 \\ c_{41} &= 354745.8 \cdot 10^3, c_{42} = 271392.08 \cdot 10^3, c_{43} = 578051.85 \cdot 10^3 \\ c_{44} &= 1071105.8 \cdot 10^3 \end{aligned}$$

From these, we calculate:

$$\begin{aligned} c_{1,\min} &= 2995769.4 \cdot 10^3, c_{1,\max} = 5145014.8 \cdot 10^3, \\ c_1^c &= 4218276.6 \cdot 10^3 \\ c_{2,\min} &= 636084 \cdot 10^3, c_{2,\max} = 1057968.8 \cdot 10^3, \\ c_2^c &= 847026.4 \cdot 10^3 \\ c_{3,\min} &= 1782624 \cdot 10^3, c_{3,\max} = 4463029 \cdot 10^3, \\ c_3^c &= 3122826.5 \cdot 10^3 \\ c_{4,\min} &= 271392.08 \cdot 10^3, c_{4,\max} = 1071105.8 \cdot 10^3, \\ c_4^c &= 671248.9 \cdot 10^3 \end{aligned}$$

Table 1. Planned and actual production

Projects	Oil [thousand tons]							
	2017		2018		2019		2020	
	Q_{ij}^{pl}	Q_{ij}^f	Q_{ij}^{pl}	Q_{ij}^f	Q_{ij}^{pl}	Q_{ij}^f	Q_{ij}^{pl}	Q_{ij}^f
AZNEFT	6327.6	6169.6	6200.0	6255.9	6346.3	6309.8	5965.8	6109.9
JV/LTDD	1311.5	1210.9	1352.1	1286.4	1336.8	1297.3	1240.0	1297.3
Gas [million m ³]								
AZNEFT	6169.6	5905.4	5520.0	5662.9	5619.3	5912.7	5913.1	5922.2
JV/LTDD	1210.9	1159.3	900.0	862.0	1199.3	1491.3	1333.9	1421.3

Q_{ij}^{pl} – in the numerator planning; Q_{ij}^f – in the denominator actual

$$c_{1min} = 2995769.4 \cdot 10^3, \quad c_{1max} = 5145014.8 \cdot 10^3,$$

$$c_1^c = 4218276.6 \cdot 10^3$$

$$c_{2min} = 636084 \cdot 10^3, \quad c_{2max} = 1057968.8 \cdot 10^3,$$

$$c_2^c = 847026.4 \cdot 10^3$$

$$c_{3min} = 1782624 \cdot 10^3, \quad c_{3max} = 4463029 \cdot 10^3,$$

$$c_3^c = 3122826.5 \cdot 10^3$$

$$c_{4min} = 271392.08 \cdot 10^3, \quad c_{4max} = 1071105.8 \cdot 10^3,$$

$$c_4^c = 671248.9 \cdot 10^3$$

$$w_1 = 537311.35 \cdot 10^3, \quad w_2 = 105471.07 \cdot 10^3,$$

$$w_3 = 670101.25 \cdot 10^3, \quad w_4 = 199928.45 \cdot 10^3$$

For $h^0 = 0.9$, the values r_i^L and r_i^R are calculated using the formulas:

$$r_i^L = c_i^c - (w_i \ln 10)^{1/2}, \quad r_i^R = c_i^c + (w_i \ln 10)^{1/2}, \quad \ln 10 \approx 2.30259.$$

Which yields:

$$r_1^L = 4217924.9 \cdot 10^3, \quad r_1^R = 4253403.6 \cdot 10^3$$

$$r_2^L = 847010.82 \cdot 10^3, \quad r_2^R = 847038.98 \cdot 10^3$$

$$r_3^L = 312274.08 \cdot 10^3, \quad r_3^R = 43122838.9 \cdot 10^3$$

$$r_4^L = 671227.45 \cdot 10^3, \quad r_4^R = 671270.35 \cdot 10^3$$

Solving the system with the values r_i^L and r_i^R ($i = 1, 2, 3, 4$) found, we obtain:

$$x_1 = 0.2703, \quad x_2 = 0.2525, \quad x_3 = 0.2301, \quad x_4 = 0.2471$$

From Table 2 for $m + 1 = 2021$ found:

$$d_{m+1}^{(oil)} = 71.6 \text{ and } d_{m+1}^{(gas)} = 846.3$$

$$\text{Then } S_{1,m+1}^{(oil)} = S_{2,m+1}^{(oil)} = 472.56, \quad S_{3,m+1}^{(gas)} = S_{4,m+1}^{(gas)} = 846.3.$$

And:

$$\tilde{S}_{1,m+1} = 5965.8 \cdot 472.56 = 2819198.5$$

$$\tilde{S}_{2,m+1} = 1240 \cdot 472.56 = 585974.4$$

$$\tilde{S}_{3,m+1} = 5913.1 \cdot 846.3 = 5004256.5$$

$$\tilde{S}_{4,m+1} = 1333 \cdot 846.3 = 1128117.9$$

Next, we calculate the total amount S_{m+1} using the following formula:

$$S_{m+1} = \sum_{i=1}^2 (Q_{i,m}^f - \tilde{Q}_{i,m}) \cdot P_{i,m+1}^{(oil)} + \sum_{i=3}^4 (Q_{i,m}^f - \tilde{Q}_{i,m}) \cdot P_{i,m+1}^{(gas)}$$

$$\text{This yields } S_{m+1} = 300574.59.$$

The additional investments in the i -project in the $(m+1)$ -th period $S_{i,m+1}^{addit}$, calculated by the ratio:

$$S_{i,m+1}^{addit} = S_{m+1} \cdot x_i, \quad (i = 1, 2, 3, 4)$$

$$S_{1,m+1}^{addit} = 7791.3, \quad S_{2,m+1}^{addit} = 44644.16,$$

$$S_{3,m+1}^{addit} = 40683.6, \quad S_{4,m+1}^{addit} = 4261.1$$

Conclusion

1. The method of modeling the minimax level of unprofitability is one of the new methods of probabilistic programming for solving the problem of investment allocation in conditions of risk associated with uncertainty and insufficient information about the income of investments in projects. It is shown that with the independence of the random variables describing returns, the problem of the investment portfolio will be reduced to a linear programming problem solved by the well-known simplex method.
2. Since in this work, by defining the loss function $r(x, c)$ for a given share vector $x = (x_1, \dots, x_n)$, $x \geq 0$, $\sum_{i=1}^n x_i = 1$ distribution of the investment vector $c = (c_1, \dots, c_n)$ for projects $i = 1, \dots, n$, the largest of the losses is minimized, the resulting solution should be the most preferable, in contrast to the concentrated solution $x_{i_0} = 1, x_i = 0, i \neq i_0$, which can be unprofitable.
3. In this paper, a method for selecting a non-negative linear combination is proposed, resulting in the problem of investment distribution being reduced to a linear programming problem, to which the simplex method is directly applicable.

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OFERTA BADAWCZA ZAKŁADU GEOLOGII I GEOCHEMII

- petrofizyczne analizy laboratoryjne: gęstość, porowatość, parametry przestrzeni porowej, analizy przepuszczalności, analiza mikro i makro szczelinowatości na płytkach cienkich i fragmentach rdzeni, petrograficzna ocena składu mineralnego, procesów diagenetycznych, analiza rozmieszczenia i geometrii przestrzeni porowej na podstawie płytek cienkich, przepływy fazowe, zwilżalność, ściślność;
- geochemiczne analizy laboratoryjne: ocena składu maceratowego i refleksyjności wityrnytu, piroliza Rock-Eval (oznaczenie zawartości TOC, wolnych węglowodorów, węgla rezydualnego), określenie potencjału węglowodorowego skał macierzystych, analiza elementarna węglowodorów i kerogenu (C,H,N,S, współczynniki H/C i D/C), skład chemiczny i izotopowy gazów, badania gazu wolnego (desorbowanego) i resztkowego, badania ekstrahowalnej substancji organicznej oraz analiza składu grupowego, analiza biomarkerów GC-MS (frakcja nasycona i aromatyczna, analiza GC-FID frakcji nasyconej), korelacja rop ze skałami macierzystymi;
- modelowanie geologiczne 3D złóż i obszarów poszukiwawczych: interpretacja danych geofizyki wiertniczej (petrofizyczna, geochemiczna, geomechaniczna, szczelinowatości), modelowanie strukturalne, facjalne i parametrów petrofizycznych z wykorzystaniem danych otworowych i sejsmicznych, modelowanie geomechaniczne, obliczanie zasobów na bazie modeli 3D złóż wraz z analizą niepewności;
- sedimentologia dla geologii naftowej: analizy sedimentologiczne rdzeni wiertniczych, analizy środowisk depozycyjnych i osadowych, stratygrafia sekwencji, analizy sedimentologiczne odstępów powierzchniowych oraz ich integracja z danymi otworowymi (rdzenie, pomiary geofizyki otworowej) i sejsmicznymi, dystrybucja rozkładu facji w basenach sedimentacyjnych, predykcja występowania ciał zbiornikowych i ich orientacja przestrzenna w skali basenu sedimentacyjnego;
- modelowanie 1D/2D/3D systemów naftowych: odtwarzanie ewolucji strukturalnej, termicznej i parametrycznej basenu sedimentacyjnego w skali czasu geologicznego, rekonstrukcja czasu i przebiegu procesów generowania i ekspulsji węglowodorów, modelowanie dróg migracji węglowodorów i miejsc ich akumulacji, ocena zasobów prognostycznych, analiza niepewności, ranking obiektów poszukiwawczych;
- zintegrowana platforma: Petrel, PetroMod, Techlog, Interactive Petrophysics, ProGeo, Petrel Reservoir Geomechanics (Visage).



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